Multi-channel ANC with Adaptive Kernel Assisted On-line Secondary Path Modeling

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***Abstract*—Conventional online secondary path modeling (SPM) for multi-channel active noise control (ANC) is often computationally intensive and unstable, particularly in large-scale systems. To address this, we propose a novel hybrid ANC method that leverages an adaptive kernel for SPM. The approach models a small subset of secondary paths in real-time and uses an adaptive kernel, whose hyperparameters are optimized via gradient descent, to accurately interpolate the remainder. Simulations in a multi-loudspeaker, multi-microphone setup demonstrate that the proposed method achieves steady-state performance comparable to conventional full-path modeling while exhibiting significantly faster and more stable convergence performance in dynamic acoustic environments.**

I. Introduction

The active noise control (ANC) technique is a highly effective method of canceling low-frequency noise by injecting anti-noise that has the same amplitude but opposite phase as the original noise [1]. Adaptive control algorithms, such as the least mean square (LMS) and its variant the filtered reference least mean square (FxLMS) algorithm [2], are widely used to tackle non-stationary noise. While single-channel ANC is now widely adopted in consumer devices (e.g., noise cancellation headphones), advancing multi-channel ANC systems to broaden noise cancellation regions remains a critical research priority [3]–[6]. For the latter, the computational complexity increases with the number of channels. Especially, the deployment of multi-channel ANC in time-varing acoustic environments remains a challenge.

In ANC systems, the secondary path refers to the acoustic propagation path from the secondary source (e.g., a loudspeaker) to the error microphone. Accurate modeling of this path is critical for achieving effective noise cancellation, as it directly influences the performance of adaptive control algorithms such as FxLMS [7]. Online secondary path modelling (Online SPM) is the most widely used technique for estimating time-varying secondary path during noise cancellation [8].

In terms of online SPM, random white noise is commonly used as an auxiliary excitation signal for secondary path identification. The core concept here involves two adaptive filters, one for ANC and the other for SPM. Obviously, injected white noise can interfere with the adaptive noise cancellation process. Therefore, various schemes have been proposed to minimize this interference. Zhang proposed an ANC system [9] that incorporates three cross-update LMS adaptive filters for the ANC controller, secondary path modeling, and interference suppression. In particular, the third adaptive filter is designed to reduce the perturbation affecting secondary path modelling and suppress the impact of the injected noise on the control filter. After that, an auxiliary noise power scheduling strategy was proposed to further mitigate the impact of injected noise [10]. Instead of introducing an additional adaptive filter, a more effective method is to estimate the secondary path through a variable step-size LMS [11], [12], that is to dynamically adjust the step size for each online secondary path modeling filter based on the power ratio of the auxiliary noise and the error signal. Recent studies have increasingly focused on addressing this challenge through a stage-based approach [13], [14] or a mode-switching strategy [15], [16],which dynamically switches between ANC and online SPM by monitoring system performance to mitigate inter-interference.

In multi-channel ANC, online SPM estimation typically requires injecting a larger amount of auxiliary noise, which can further compromise noise control performance [17]. To mitigate this issue, hybrid ANC frameworks with power-controlled SPM subsystems and signal decomposition strategies have been proposed, enabling dynamic trade-offs between modeling accuracy and noise injection [18]. Other studies have explored adaptive filtering architectures that incorporate decorrelation techniques and gain-modified algorithms to improve modeling reliability while maintaining computational efficiency [19]. In addition, integration with virtual sensing and spatial interpolation methods such as kriging has enabled accurate sound field estimation in the absence of physical microphones, further enhancing the adaptability of ANC systems [20].

In this paper, we propose an adaptive-kernel-assisted online SPM strategy for multichannel ANC. Unlike conventional approaches that require full estimation of all secondary paths, our method selectively estimates a small subset of channels during online SPM and leverages neural network (NN)-based adaptive kernel methods to reconstruct the remaining channels from the sparse estimates. Through experimental simulations, we demonstrate that the proposed method achieves steady-state performance comparable to conventional full-path modeling

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while exhibiting significantly faster and more stable noise cancellation performance in time-varying environments.

II. Review: Multi-Channel ANC with Online SPM

We firstly review the conventional multi-channel ANC system that employs the FxLMS adaptive algorithm for noise cancellation and online SPM using auxilinary white noise. As shown in Fig. 1, this framework integrates adaptive filtering and real-time path estimation to enhance noise cancellation performance in dynamic environments.

More specifically, we consider a multichannel ANC system with one reference sensor, L secondary sources, and M microphones. The reference signal for the primary noise is denoted as $x(n)$, where n represents the discrete-time index. A segment of I samples of the reference signal is represented by a vector $\mathbf{x}_I(n) = [x(n) \quad x(n-1) \quad \cdots \quad x(n-I+1)]^T$. The superscript ^{T} denotes the transpose operator. The vector of $\mathbf{w}_\ell(n), \ell = 1, ..., L$ represents the tap weights of the ℓth secondary source, that is, $\mathbf{w}_\ell(n) = [w_\ell(0) \quad w_\ell(1) \quad \cdots \quad w_\ell(I-1)]^T$, which are adaptively adjusted to generate anti-noise signals. The output signal of the ℓth secondary source is written as

$$y_\ell(n) = \mathbf{w}_\ell^T(n)\mathbf{x}_I(n) + v_\ell(n), \tag{1}$$

where the first component on the right hand side is dedicated to noise cancellation, while the second component denotes the auxilinary white noise for online SPM.

The acoustic response from the ℓth secondary source to the mth microphone is represented by an FIR filter $\mathbf{h}_{\ell m}$ of length J, that is, $\mathbf{h}_{\ell m} = [h_{\ell m}(0) \quad h_{\ell m}(1) \quad \cdots \quad h_{\ell m}(J-1)]^T$. Then, the residual noise signal at the mth microphones is written as

$$\begin{aligned} e_m(n) &= d_m(n) + \sum_{\ell=1}^{L}[\mathbf{h}_{\ell m}^T\mathbf{y}_\ell(n) + \mathbf{h}_{\ell m}^T\mathbf{v}_\ell(n)] \\ &= d_m(n) + \sum_{\ell=1}^{L}[\mathbf{h}_{\ell m}^T\mathbf{y}_\ell(n)] + \sum_{\ell=1}^{L}[\mathbf{h}_{\ell m}^T\mathbf{v}_\ell(n)] \\ &= d_m(n) + y'_m(n) + v'_m(n), m = 1, ..., M, \end{aligned} \tag{2}$$

where $d_m(n)$ denotes the primary noise signal at the mth microphone. $\mathbf{y}_\ell(n) = [y_\ell(n) \quad y_\ell(n-1) \quad \cdots \quad y_\ell(n-J+1)]^T$ represents the secondary noise signal from the ℓth secondary source, and $\mathbf{v}_\ell(n) = [v_\ell(n) \quad v_\ell(n-1) \quad \cdots \quad v_\ell(n-J+1)]^T$ is the white noise injected for secondary path modelling. $y'_m(n)$ and $v'_m(n)$ denotes the secondary noise and modeling signal received at the mth microphone, respectively.

For noise cancellation purpose, the residual noise in (2) should be minimized. Thus, the coefficients of the $\mathbf{w}_\ell(n)$ are updated using the FxLMS algorithm

$$\begin{aligned} \mathbf{w}_\ell(n+1) &= \mathbf{w}_\ell(n) - \mu_w \sum_{m=1}^{M}[e_m(n)\hat{\mathbf{x}}'_{\ell m}(n)] \\ &= \mathbf{w}_\ell(n) - \mu_w \sum_{m=1}^{M}[(d_m(n) + y'_m(n))\hat{\mathbf{x}}'_{\ell m}(n)] \\ &\quad - \mu_w \sum_{m=1}^{M}[v'_m(n)\hat{\mathbf{x}}'_{\ell m}(n)], \end{aligned} \tag{3}$$

where μ_w is the step size of the noise controlling process, $\hat{x}'_{\ell m}(n) = \mathbf{x}_J^T(n)\hat{\mathbf{h}}_{\ell m}(n)$ represents the reference signal filtered by the estimated second path $\hat{\mathbf{h}}_{\ell m}(n)$, and $\mathbf{x}_J(n) = [x(n) \quad x(n-1) \quad \cdots \quad x(n-J+1)]^T$, $\hat{\mathbf{x}}'_{\ell m}(n) = [\hat{x}'_{\ell m}(n) \quad \hat{x}'_{\ell m}(n-1) \quad \cdots \quad \hat{x}'_{\ell m}(n-I+1)]^T$.

Then for online SPM, the estimates of the acoustic response $\hat{\mathbf{h}}_{\ell m}$ are updated using the LMS algorithm,

$$\begin{aligned} \hat{\mathbf{h}}_{\ell m}(n+1) &= \hat{\mathbf{h}}_{\ell m}(n) + \mu_h[\hat{v}'_m(n) - e_m(n)]\mathbf{v}_\ell(n) \\ &= \hat{\mathbf{h}}_{\ell m}(n) + \mu_h[\hat{v}'_m(n) - \hat{v}_m(n)]\mathbf{v}_\ell(n) \\ &\quad - \mu_h[d_m(n) + y'_m(n)]\mathbf{v}_\ell(n), \end{aligned} \tag{4}$$

where μ_h is the step size of the secondary path modeling process, $\hat{v}'_m(n) = \sum_{\ell=1}^{L}\mathbf{v}_\ell(n)^T\hat{\mathbf{h}}_{\ell m}(n)$ represents the estimate modeling signal $v'_m(n)$.

As shown in (3) and (4), the noise cancelation process and the online SPM process interfere with each other. In addition, in the multi-channel ANC, the computational load for online secondary path estimation increases significantly with the number of channels to be estimated.

III. Adaptive Kernel Assisted On-line Seconday Path Modeling

In this section, a multi-channel ANC system with an adaptive kernel-assisted online SPM strategy is proposed. As shown in Fig. 1, the approach involves estimating a sparse subset of secondary paths using auxilinary white noise during online operation. The remaining secondary paths are then reconstructed from these sparse estimates using an NN-based adaptive kernel interpolation method [21].

A. Kernel Ridge Regression

Kernel methods have been widely used for sound field reconstruction and estimation. The objective is to estimate a sound field $u(\mathbf{r}, k)$ at wave number $k = 2\pi f/c$ for any spatial point $\mathbf{r} \in \Omega$ within a source-free region from a finite set of measurements $\{s_m\}_{m=1}^{M}$. Here, f denotes the frequency, c refers to the speed of sound. We omit the dependency k for notational simplicity and denote sound field as $u(\mathbf{r})$.

A sound field $u(\mathbf{r})$ that satisfies the Helmholtz equation can be decomposed as a superposition of plane waves. This decomposition induces a reproducing kernel Hilbert space (RKHS) $\mathcal{H}$, for which a reproducing kernel function $\kappa(\cdot, \cdot)$ exists. Based on kernel ridge regression, the estimation of the sound field $\hat{u}(\mathbf{r})$ can be represented with kernel function $\kappa(\cdot, \cdot)$ as

$$\hat{u}(\mathbf{r}) = \boldsymbol{\kappa}(\mathbf{r})\boldsymbol{\alpha}, \tag{5}$$

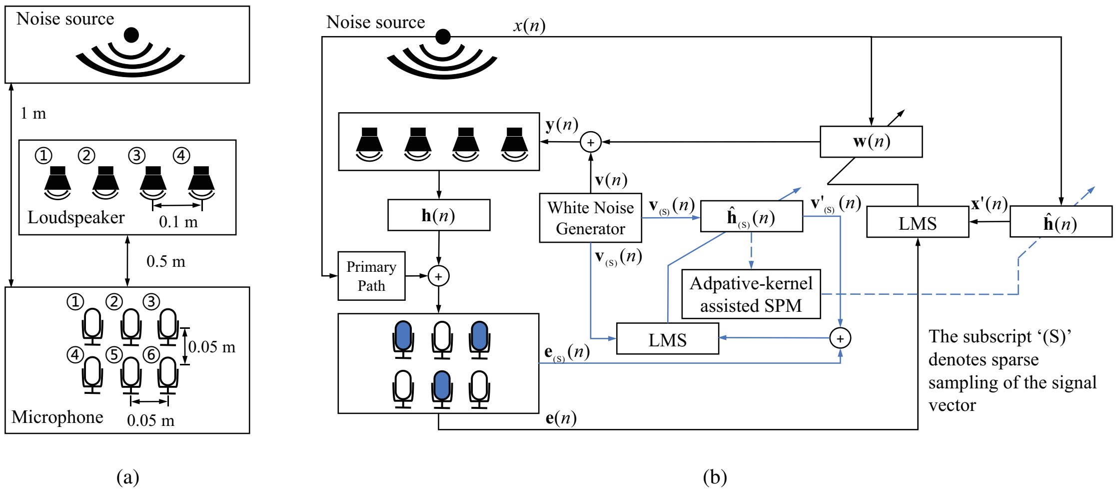

Fig. 1. System set-up and block diagram of the proposed method: (a) An example of the system setup with 4 loudspeakers and 6 microphones. (b) Block diagram of the proposed ANC system with adaptive-kernel assisted online SPM.The blue markers and lines indicate sparse sampling of the signal vector.

where

$$\boldsymbol{\kappa}(\mathbf{r}) = \left[\ \kappa(\mathbf{r}, \mathbf{r}_1) \quad \ldots \quad \kappa(\mathbf{r}, \mathbf{r}_M)\ \right], \tag{6}$$

$$\boldsymbol{\alpha} = (\mathbf{K} + \lambda \mathbf{I})^{-1}\mathbf{s}. \tag{7}$$

Here, $\lambda > 0$ is a regularization constant, $\mathbf{s} = [s_1, \ldots, s_M]^T$ is the measurement vector, $\boldsymbol{\kappa}$ is the function vector, and $\mathbf{K}$ is defined as

$$\mathbf{K} = \begin{bmatrix} \kappa(\mathbf{r}_1, \mathbf{r}_1) & \ldots & \kappa(\mathbf{r}_1, \mathbf{r}_M) \\ \vdots & \ddots & \vdots \\ \kappa(\mathbf{r}_M, \mathbf{r}_1) & \ldots & (\mathbf{r}_M, \mathbf{r}_M) \end{bmatrix}. \tag{8}$$

The selection of an appropriate kernel function is critical for optimizing SPM performance. For instance, plane wave kernels excel at representing directional plane wavefronts, while Bessel kernels are well-suited for modeling diffusive or reverberant fields. However, fixed-form kernels lack adaptability to complex and dynamic environments (e.g., hybrid directional-diffuse scenarios in room impulse response modelling).

B. NN-based Adaptive Kernels for Secondary Path Estimation

We adopt the same modeling strategy as proposed in [21]. First, we assume that within a room environment, the secondary path from the secondary source to the error microphone comprises a directional component with stronger energy and a residual component with weaker energy. Therefore, the kernel κ is decomposed into two parts, i.e., the directional kernel κ_{dir} and the residual kernel κ_{res} written as

$$\kappa = \kappa_{\text{dir}} + \kappa_{\text{res}}. \tag{9}$$

The directional kernel is modelled as a combination of a set of sparse, high-amplitude oriented kernels, each of which is estimated to be biased towards a specific direction. By modeling the source directional statistics using the von Mises-Fisher distribution [22], the directional kernel function κ_{dir} has a closed-form solution as

$$\kappa_{\text{dir}}(\mathbf{r}, \mathbf{r}') = \sum_{n=1}^{N} \gamma_n \frac{j_0\left(\sqrt{[k(\mathbf{r}-\mathbf{r}') - \mathrm{i}\beta_n \mathbf{d}_n]^T [k(\mathbf{r}-\mathbf{r}') - \mathrm{i}\beta_n \mathbf{d}_n]}\right)}{C(\beta_n)}. \tag{10}$$

where

$$C(\beta) = \begin{cases} 1, & \beta = 0, \\ \frac{\sinh(\beta)}{\beta}, & \text{otherwise} \end{cases}, \tag{11}$$

$$\boldsymbol{\beta} = [\beta_1, \beta_2, \ldots, \beta_N]^T \tag{12}$$

$$\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \ldots, \gamma_N]^T. \tag{13}$$

Here, $j_0(\cdot)$ is the zeroth-order spherical Bessel function, $\mathbf{d}_n$ is the bias directions, N is the number of bias directions, $\boldsymbol{\beta}$ are the amplitude coefficients relating to each bias direction, and $\boldsymbol{\gamma}$ are the combination weights of the various von MisesFisher distributions.

Due to the low amplitude of the residual sound field and its complicated interaction with the environment, a neural network is used to model the residual kernel, enabling it to learn complex patterns from the data. That is,

$$\kappa_{\text{res}} = \mathcal{T}(w_{\text{res}})$$
$$w_{\text{res}}(\boldsymbol{\eta}; \boldsymbol{\theta}) = \text{NN}(\boldsymbol{\eta}; \boldsymbol{\theta}), \tag{14}$$

where $\mathcal{T}$ is a numerical approximation of the integral, $\boldsymbol{\eta} = [\mathbf{d}_1, \ldots, \mathbf{d}_N]^T$ is the bias direction vector, $w_{\text{res}}(\boldsymbol{\eta}; \boldsymbol{\theta})$ is the weight function specific to the given directional kernel distribution $\boldsymbol{\eta}$, $\boldsymbol{\theta}$ is the set of NN parameters that need to be trained. The kernel function κ_{res} is approximated by numerical integration.

Assume that the positions of the M error microphones are represented as $\{\mathbf{r}_m\}_{m=1}^{M}$, where the secondary paths corresponding to the M_1 microphones are estimated during the online SPM process, and the objective here is to reconstruct the remaining secondary paths. M_1 should be much smaller than M. Define $\{\hat{\mathbf{H}}_{\ell m}(k)\}_{m=1}^{M}$ as the frequency-domain transfer function of the secondary path $\{\hat{\mathbf{h}}_{\ell m}(n)\}_{m=1}^{M}$. Next, we denote the subset of secondary paths that are estimated during the online operation using (4) as $\hat{\mathbf{H}}_{\mathrm{E}} = [\hat{\mathbf{H}}_{\ell 1}(k), \ldots, \hat{\mathbf{H}}_{\ell M_1}(k)]^T$. The remaining secondary paths to be reconstructed are denoted as $\hat{\mathbf{H}}_{\mathrm{R}} = [\hat{\mathbf{H}}_{\ell (M_1+1)}(k), \ldots, \hat{\mathbf{H}}_{\ell M}(k)]^T$.

We apply the NN-based adaptive kernel function to represent the frequency-domain secondary paths. Given the online estimated secondary paths $\hat{\mathbf{H}}_{\mathrm{E}}$, the loss function is defined as

$$\mathcal{L}(u) = \sum_{m=1}^{M_1} \left| \hat{\mathbf{H}}_{\ell m} - u(\mathbf{r}_m) \right|^2 + \lambda \|u\|_{\mathcal{H}}^2. \tag{15}$$

Therefore, the optimization problem for obtaining the optimal parameters is defined as

$$\begin{aligned}\mathcal{L}_{\text{opt}}(\boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\theta}) = & \|\hat{\mathbf{H}}_{\mathrm{E}} - \mathbf{K}(\mathbf{K} + \lambda \mathbf{I})^{-1} \hat{\mathbf{H}}_{\mathrm{E}}\|_2^2 \\ & + \lambda \hat{\mathbf{H}}_{\mathrm{E}}^T (\mathbf{K} + \lambda \mathbf{I})^{-1} \mathbf{K} (\mathbf{K} + \lambda \mathbf{I})^{-1} \hat{\mathbf{H}}_{\mathrm{E}}.\end{aligned} \tag{16}$$

The optimal parameters $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$ and $\boldsymbol{\theta}$ are obtained by minimizing (17) through the gradient descent method. The partial derivatives can be expressed as

$$\frac{\partial \mathcal{L}_{\text{opt}}}{\partial \boldsymbol{\chi}} = -\lambda \hat{\mathbf{H}}_{\mathrm{E}}^T (\mathbf{K} + \lambda \mathbf{I})^{-1} \frac{\partial \mathbf{K}}{\partial \boldsymbol{\chi}} (\mathbf{K} + \lambda \mathbf{I})^{-1} \hat{\mathbf{H}}_{\mathrm{E}}, \tag{17}$$

where $\boldsymbol{\chi}$ is a given kernel parameter to calculate the gradient.

Once the optimized parameters are calculated, the adaptive kernel κ in (10) is constructed, and the remaining channels $\hat{\mathbf{H}}_{\mathrm{R}}$ are reconstructed using (5).

IV. Simulation Experiments and Results

A. Experimental Settings

We consider an experiment setup as in Fig. 1 with 4 loudspeakers and 6 microphones. The RIRs (Room Impulse Responses) are generated using the image source method [23]. A shoebox-shaped room (8 m × 8 m × 3 m) is simulated with a reverberation time of 0.4 s. The microphone array is placed in the center of the room, with a spacing of 0.05 m between each element. The loudspeaker array with 0.1 m spacing is placed 0.5 m from the center of the microphone array. The primary noise source is placed 1 m from the center of the microphone array. The speed of sound is set to 340 m/s. The sampling rate is 2kHz and all RIRs were truncated to 64 ms (the tap weight length of the secondary path is 128). The initial secondary path model was determined through an offline, iterative estimation process, which was terminated once the estimation error converged to -5 dB.

The reference singal is a composite of four sinusoids at 100, 200, 300 and 400 Hz. Each component has an independent random phase, and the signal is normalized to a variance of 2. The SPM excitation signal is generated by filtering four mutually uncorrelated, zero-mean Gaussian white noise sequences (each with a variance of 0.02) through a 50-450 Hz bandpass filter. This process yields an accurate SPM with spectral properties

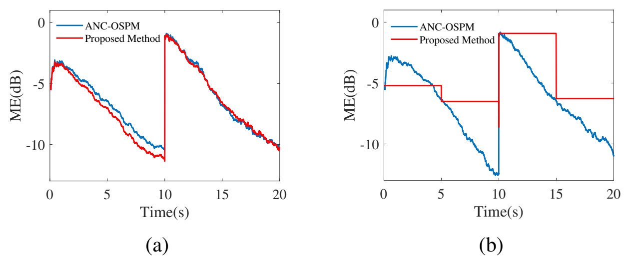

Fig. 2. ME for (a) a directly online estimated path between Loudspeaker 2 and Microphone 3 and (b) a kernel interpolated path between Loudspeaker 2 and Microphone 4. The proposed method (red line) is compared against the conventional full-path online modeling approach (blue line).).

aligned with those of the target noise. The step sizes of adaptive filters are adjusted to achieve fast and stable convergence, that is for online SPM, $\mu_w = 4 \times 10^{-5}$, $\mu_h = 0.025$; for proposed methed, $\mu_w = 5 \times 10^{-5}$, $\mu_h = 0.025$. The proposed method implements online modeling to microphones 1, 3, and 5, and uses offline interpolation for Microphones 2, 4, and 6. The simulation is conducted over 20 seconds, with an abrupt secondary path variation introduced at the 10-second point, triggered by a relocation of the louspeakers. The adpative-kernel assisted SPM system updates the secondary path every 5 seconds. The neural network architecture of adaptive-kernel is the seem as described in [21].

B. Simulation results

This section evaluates the performance of the proposed method in simulation experiments. The SPM accuracy is evaluated by the relative modeling error between the estimated $\hat{\mathbf{h}}_{\ell m}$ and true $\mathbf{h}_{\ell m}$ secondary paths, calculated as follows

$$ME(n) = \frac{\|\mathbf{h}_{\ell m}(n) - \hat{\mathbf{h}}_{\ell m}(n)\|_2^2}{\|\mathbf{h}_{\ell m}(n)\|_2^2} \tag{18}$$

Notably, the performance evaluation is limited to the 50-450 Hz spectral range of the primary noise. The noise reduction performance is quantified using the following metric

$$NR(n) = \frac{\sum_{m=1}^{M} e_m^2(n)}{\sum_{m=1}^{M} d_m^2(n)}, \tag{19}$$

Fig. 2 illustrates the evolution of the online secondary path modeling error (ME) in the iterative process. To provide a clearer comparison between the conventional and proposed methods, the modeling error of one secondary path is selected for analysis, while the remaining paths exhibit similar trends. As shown in the figure, for the secondary paths updated via online modeling, both the conventional and proposed methods achieve comparable accuracy. For the secondary path updated offline, the results demonstrate that the proposed method, which uses NN-based interpolation for offline paths, achieves a final convergence accuracy equivalent to that of a conventional full online modeling approach.

As shown in Fig. 3, given an initial secondary path model with a -5 dB estimation error, the proposed method converges at nearly the same rate as the conventional full-path modeling

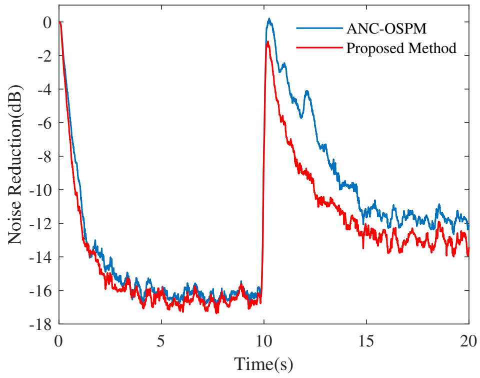

Fig. 3. Noise reduction at the microphone locations achieved by the conventional ANC-OPSM method and the proposed method.

approach, and reached the steady-state performance of the conventional approach. It can be observed that performing online modeling for all secondary paths impairs the system's convergence rate. Although the proposed method only models a subset of the paths, it effectively mitigates the adverse impact of online modeling on the system. Following an abrupt path change, the proposed method exhibits more robust and rapid reconvergence, comprared to the proposed approach. This demonstrates that the proposed hybrid online/offline strategy prevents the large modeling errors that degrades the performance of an online, full-path system.

V. Conclusion

In this paper, we introduced a multi-channel active noise control system that utilizes an adaptive kernel to assist with online secondary path modeling. Our proposed hybrid online/offline strategy was shown to effectively balance modeling accuracy and robustness. Especially, simulations demonstrated that upon an abrupt change in the acoustic environment, our method exhibits superior convergence performance, converging faster and more smoothly than the traditional approach.

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