# Directional Filtering of Sound Fields for Emphasizing Specific Directions of Arrival and Its Applications

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***Abstract*—This paper proposes a directional filtering method for sound fields that enhances components arriving from a specified direction while preserving spatial acoustic information. The proposed directional filtering is formulated as a linear operator that applies directional weighting to a sound field based on the direction of arrival. Under a von Mises–Fisher-type weighting function, we derive a closed-form expression of the proposed directional filtering for sound fields represented as a superposition of spherical wave functions, which is one of the fundamental representations of sound fields. As an application, we also present a framework in which the proposed directional filtering is applied to sound fields estimated using microphone array measurements. Numerical experiments demonstrate that the proposed directional filtering can effectively enhance a specified directional component, even when the original sound field is not represented in an object-based manner, including source position information.**

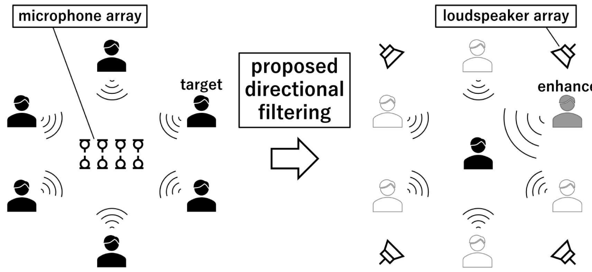


Fig. 1: Application example of the proposed directional filtering. In a system that reproduces sound fields captured by microphones using a loudspeaker array, the proposed directional filtering can be employed as an intermediate processing step to enable enhancement of components from a specified direction.

## I. Introduction

In recent years, the quality of spatial acoustic experiences has become increasingly important in applications such as augmented reality (AR), virtual reality (VR), and immersive media that utilize spatial audio technologies [1]–[5]. Representative capture and reproduction methods for highly immersive spatial audio systems include sound field measurement using microphone arrays [6], [7], sound field reproduction using loudspeaker arrays [6]–[8], and binaural reproduction using earphones or headphones [9]–[11]. When applying these capture and reproduction technologies in AR/VR environments, it is often desirable not only to acquire and reproduce sound fields but also to selectively emphasize or suppress specific audio components according to user preferences. In particular, spatial filtering based on the direction of arrival is useful in many scenarios, as illustrated in Fig. 1.

Beamforming is one of the commonly used signal processing methods for emphasizing or suppressing sound components based on their direction of arrival [12], [13]. It enhances sounds arriving from a specific direction while suppressing those from other directions by controlling the directional sensitivity of a microphone array. However, most beamforming methods are formulated as a multiple-input single-output process, and the resulting output signal does not retain spatial acoustic information, such as the distance and directionality of the sound source. Although research on binaural beamforming has progressed [13], it remains challenging to adapt to dynamic changes in the user's position and orientation.

To address this limitation from a new perspective, we propose a method for directional filtering of sound fields. Specifically, we formulate the directional filtering process as an operator that maps a sound field to a modified sound field in which components arriving from a specified direction are emphasized. Although the proposed directional filtering is formulated in terms of a directional integral, we derive its closed-form expression, under a von Mises–Fisher-type weighting function, for sound fields represented as a superposition of spherical wave functions, which is one of the fundamental representations of sound fields. Since the proposed directional filtering processes sound fields as both input and output, it can be combined with the aforementioned sound field capture and reproduction techniques. This integration is expected to enable technologies that have been challenging until now, such as reproducing captured sound fields through earphones or speakers with enhanced components from specific directions, even for a moving listener. As an application example, we present a framework where the proposed directional filtering is applied to sound fields estimated from microphone array measurements, along with a derivation of a closed-form expression for the resulting filtered field. The effectiveness of the proposed directional filtering is quantitatively evaluated

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through numerical experiments, where the proposed directional filtering was applied to sound fields estimated using a microphone array. In this scenario, although the estimated sound field is no longer represented in an object-based manner that includes source position information, we demonstrate that the proposed directional filtering can effectively enhance components arriving from a specified direction.

## II. Proposed Method

As a preliminary to the proposed method, we consider a sound field $u$ defined in a simply connected open set $\Omega \subset \mathbb{R}^3$ under a given angular frequency $\omega$. When no sound sources exist within $\Omega$, the sound field at any point $\boldsymbol{r} \in \Omega$ satisfies the following Helmholtz equation in the frequency domain [14]–[16]:

$$(\Delta + k^2)u(\boldsymbol{r}) = 0, \tag{1}$$

where $\Delta$ denotes the Laplace operator, $k := \omega/c$ is the wavenumber, and $c$ is the speed of sound. A typical representation of the solution to (1) is the superposition of plane waves given by

$$u(\boldsymbol{r}) = \int_{\boldsymbol{x}\in\mathbb{S}_2} \tilde{u}(\boldsymbol{x}) \exp(-ik\boldsymbol{x}\cdot\boldsymbol{r})\mathrm{d}S, \tag{2}$$

where $\mathbb{S}_2$ is the unit sphere in $\mathbb{R}^3$, and $\tilde{u}(\boldsymbol{x}) \in \mathbb{C}$ denotes the complex amplitude of a plane wave arriving from direction $\boldsymbol{x} \in \mathbb{S}_2$. We define the function space $\mathcal{H}$ consisting of all sound fields representable by (2) as

$$\mathcal{H} := \left\{ \int_{\boldsymbol{x}\in\mathbb{S}_2} \tilde{u}(\boldsymbol{x}) \exp(-ik\boldsymbol{x}\cdot\boldsymbol{r})\mathrm{d}S \;\middle|\; \tilde{u} \in L_2(\mathbb{S}_2) \right\}, \tag{3}$$

where $L_2(\mathbb{S}_2)$ denotes the set of square-integrable functions on $\mathbb{S}_2$.

In this study, we propose a sound field directional filtering method that emphasizes components arriving from a desired direction by applying directional filtering to sound fields represented by (2). We define an operator $L_w$ that applies such directional filtering as follows:

$$(L_w u)(\boldsymbol{r}) := \int_{\boldsymbol{x}\in\mathbb{S}_2} w(\boldsymbol{x})\tilde{u}(\boldsymbol{x}) \exp(-ik\boldsymbol{x}\cdot\boldsymbol{r})\mathrm{d}S, \tag{4}$$

where $w(\boldsymbol{x})$ is a weighting function applied to plane waves arriving from direction $\boldsymbol{x}$. Although various forms of $w(\boldsymbol{x})$ are theoretically possible, in this study we consider the following function, defined by a weighting parameter $\beta \in [0, \infty)$ and a target direction vector $\boldsymbol{\eta} \in \mathbb{S}_2$:

$$w(\boldsymbol{x}) := \frac{\exp(\beta\boldsymbol{x}\cdot\boldsymbol{\eta})}{\exp(\beta)}. \tag{5}$$

This function $w$ is also known, up to a normalization constant, as the probability density function of the von Mises–Fisher distribution used in directional statistics. In particular, $w(\boldsymbol{x})$ attains its maximum value of 1 when $\boldsymbol{x} = \boldsymbol{\eta}$, indicating that the greatest weight is assigned to plane waves arriving from direction $\boldsymbol{\eta}$. This property ensures that the operator $L_w$ enhances components from the specified direction.

In general, evaluating the integral in (4) in closed form for an arbitrary $u \in \mathcal{H}$ is challenging. Therefore, we consider applying the directional filtering to sound fields expressed via a finite-order expansion using the spherical wave functions [7], [17], given by

$$u(\boldsymbol{r}) = \sum_{\nu=0}^{N} \sum_{\mu=-\nu}^{\nu} \mathring{u}_{\nu,\mu}(\boldsymbol{r}_0)\xi_{\nu,\mu}(k(\boldsymbol{r}-\boldsymbol{r}_0)), \tag{6}$$

where $\mathring{u}_{\nu,\mu}(\boldsymbol{r}_0) \in \mathbb{C}$ and $\xi_{\nu,\mu}(\cdot)$ denote the expansion coefficients and the spherical wave functions of order $\nu$ and degree $\mu$, respectively [18]. The spherical wave function is defined as

$$\xi_{\nu,\mu}(\boldsymbol{z}) := \frac{1}{i^\nu} j_\nu\left(\|\boldsymbol{z}\|\right) \hat{Y}_{\nu,\mu}\left(\frac{\boldsymbol{z}}{\|\boldsymbol{z}\|}\right), \tag{7}$$

where $\boldsymbol{z} \in \mathbb{R}^3$, $\|\cdot\|$ denotes the Euclidean norm, $j_\nu(\cdot)$ is the spherical Bessel function of the first kind of order $\nu$ [19], and $\hat{Y}_{\nu,\mu}(\cdot)$ is the unnormalized spherical harmonic function of order $\nu$ and degree $\mu$ [15], defined using the normalized spherical harmonic function $Y_{\nu,\mu}(\cdot)$ [19] as

$$\hat{Y}_{\nu,\mu}(\boldsymbol{x}) := \sqrt{4\pi}Y_{\nu,\mu}(\boldsymbol{x}). \tag{8}$$

For a sound field $u$ represented as in (6), $L_w u$ can be derived in closed form as follows. First, using the relationship [18]

$$\xi_{\nu,\mu}(\boldsymbol{z}) = \frac{1}{4\pi}\int_{\boldsymbol{x}\in\mathbb{S}_2} \exp(-i\boldsymbol{z}\cdot\boldsymbol{x})\hat{Y}_{\nu,\mu}(\boldsymbol{x})\mathrm{d}S, \tag{9}$$

the expansion in (6) can be rewritten as a superposition of plane waves:

$$u(\boldsymbol{r}) = \int_{\boldsymbol{x}\in\mathbb{S}_2} \frac{1}{4\pi} \sum_{\nu=0}^{N} \sum_{\mu=-\nu}^{\nu} \mathring{u}_{\nu,\mu}(\boldsymbol{r}_0)\hat{Y}_{\nu,\mu}(\boldsymbol{x}) \cdot \exp(-ik\boldsymbol{x}\cdot(\boldsymbol{r}-\boldsymbol{r}_0))\mathrm{d}S. \tag{10}$$

From (4), (5), and (10), we obtain

$$(L_w u)(\boldsymbol{r}) = \int_{\boldsymbol{x}\in\mathbb{S}_2} \frac{1}{4\pi\exp(\beta)} \sum_{\nu=0}^{N} \sum_{\mu=-\nu}^{\nu} \mathring{u}_{\nu,\mu}(\boldsymbol{r}_0)\hat{Y}_{\nu,\mu}(\boldsymbol{x}) \cdot \exp(-ik\boldsymbol{x}\cdot(\boldsymbol{r}-\boldsymbol{r}_0) + i\beta\boldsymbol{\eta}\cdot\boldsymbol{x})\mathrm{d}S. \tag{11}$$

Here, it is shown in [18] that (9) remains valid even when the definition of the spherical wave function is extended to the complex domain as follows:

$$\xi_{\nu,\mu}(\boldsymbol{z}) := \frac{1}{i^\nu} j_\nu\left((z_1^2+z_2^2+z_3^2)^{\frac{1}{2}}\right) \hat{y}_{\nu,\mu}\left(\frac{\boldsymbol{z}}{(z_1^2+z_2^2+z_3^2)^{\frac{1}{2}}}\right), \tag{12}$$

where $\boldsymbol{z} = [z_1, z_2, z_3]^\mathsf{T} \in \mathbb{C}^3$, and $\hat{y}_{\nu,\mu}(\cdot)$ is the homogeneous harmonic polynomial satisfying $\hat{y}_{\nu,\mu}(\boldsymbol{x}) = \hat{Y}_{\nu,\mu}(\boldsymbol{x})$ for $\boldsymbol{x} \in \mathbb{S}_2$. The expression $(z_1^2+z_2^2+z_3^2)^{\frac{1}{2}}$ denotes the complex square root with its branch cut along the negative real axis; note that it differs from the standard norm $(|z_1|^2+|z_2|^2+|z_3|^2)^{\frac{1}{2}}$ for complex values. Substituting $\boldsymbol{z}=k\boldsymbol{r}+i\beta\boldsymbol{\eta}$ into (9) yields

$$\xi_{\nu,\mu}(k\boldsymbol{r}+i\beta\boldsymbol{\eta}) = \frac{1}{4\pi}\int_{\boldsymbol{x}\in\mathbb{S}_2}\exp(-i(k\boldsymbol{r}+i\beta\boldsymbol{\eta})\cdot\boldsymbol{x})\hat{Y}_{\nu,\mu}(\boldsymbol{x})\mathrm{d}S. \tag{13}$$

Therefore, from (11) and (13), we finally obtain the following closed-form expression:

$$(L_w u)(\boldsymbol{r}) = \frac{1}{\exp(\beta)}\sum_{\nu=0}^{N}\sum_{\mu=-\nu}^{\nu}\mathring{u}_{\nu,\mu}(\boldsymbol{r}_0)\xi_{\nu,\mu}(k(\boldsymbol{r}-\boldsymbol{r}_0)+i\beta\boldsymbol{\eta}). \tag{14}$$

Since many spatial-audio techniques, including higher order Ambisonics, are based on the modeling of sound fields as superpositions of the spherical wave functions [7], this result demonstrates that the proposed directional filtering is widely applicable to such sound field representations.

## III. Application of the Proposed Method

As an application example, we present a framework in which the proposed directional filtering is applied to a sound field estimated using a microphone array, on the basis of the sound field estimation method proposed in [18], [20], [21]. We begin by briefly summarizing the sound field estimation method [18], [20], [21]. Suppose that $M$ omnidirectional microphones are placed at positions $\boldsymbol{r}_1,\cdots,\boldsymbol{r}_M\in\Omega$, and that the sound field $u$ is estimated from their observed signals $s_1,\cdots,s_M\in\mathbb{C}$. In this case, the estimated sound field $\hat{u}$ can be obtained by solving the following optimization problem:

$$\underset{u\in\mathcal{H}}{\text{minimize}}\quad Q(u) := \sum_{m=1}^{M}|u(\boldsymbol{r}_m)-s_m|^2+\lambda\|u\|_{\mathcal{H}}^2. \tag{15}$$

Here, $\lambda\in(0,\infty)$ is the regularization parameter, and $\|\cdot\|_{\mathcal{H}}$ denotes the norm on $\mathcal{H}$ defined as $\|u\|_{\mathcal{H}}^2=4\pi\int_{\boldsymbol{x}\in\mathbb{S}_2}|\tilde{u}(\boldsymbol{x})|^2\mathrm{d}S$. By solving the optimization problem (15), $\hat{u}$ is represented as a superposition of the spherical wave functions as

$$\hat{u}(\boldsymbol{r}) = \sum_{m=1}^{M}\hat{\alpha}_m v_m(\boldsymbol{r}), \tag{16}$$

where $v_m(\cdot)$ is defined as

$$v_m(\boldsymbol{r}) = \xi_{0,0}(k(\boldsymbol{r}-\boldsymbol{r}_m)). \tag{17}$$

The coefficient vector $\hat{\boldsymbol{\alpha}} := [\hat{\alpha}_1,\cdots,\hat{\alpha}_M]^{\mathsf{T}}\in\mathbb{C}^M$ is given by

$$\hat{\boldsymbol{\alpha}} = (\boldsymbol{K}+\lambda\boldsymbol{I})^{-1}\boldsymbol{s}. \tag{18}$$

Here, $\boldsymbol{I}\in\mathbb{C}^{M\times M}$ denotes the $M\times M$ identity matrix, and $\boldsymbol{s}\in\mathbb{C}^M$ and $\boldsymbol{K}\in\mathbb{C}^{M\times M}$ are defined as follows:

$$\boldsymbol{s} := [s_1,\cdots,s_M]^{\mathsf{T}}, \tag{19}$$

$$\boldsymbol{K} := \begin{bmatrix} K_{1,1} & \cdots & K_{1,M} \\ \vdots & \ddots & \vdots \\ K_{M,1} & \cdots & K_{M,M} \end{bmatrix}. \tag{20}$$

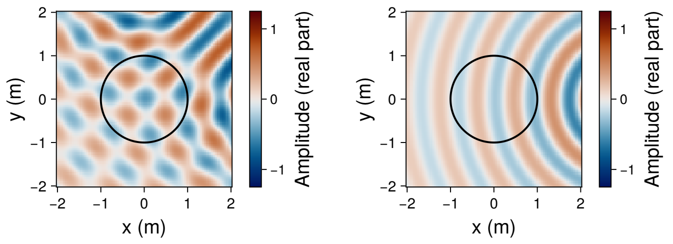



Fig. 2: Real part of the sound fields on the $xy$-plane ($z=0$): (a) mixed sound field; (b) target sound field.

Each element $K_{m_1,m_2}$ is defined by

$$K_{m_1,m_2} := \xi_{0,0}(k(\boldsymbol{r}_{m_1}-\boldsymbol{r}_{m_2})). \tag{21}$$

The proposed directional filtering can also be applied to sound fields obtained in the above manner. By the linearity of $L_w$, applying $L_w$ to each term on the right-hand side of (16), using (14), yields the following expression:

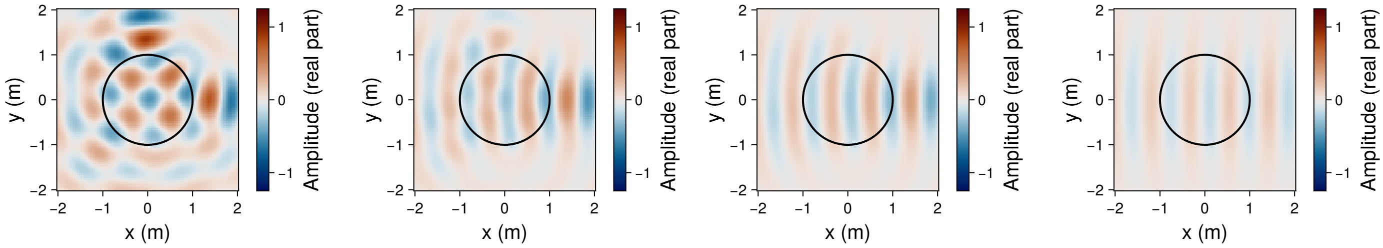
$$(L_w\hat{u})(\boldsymbol{r}) = \sum_{m=1}^{M}\hat{\alpha}_m(L_w v_m)(\boldsymbol{r}) = \frac{1}{\exp(\beta)}\sum_{m=1}^{M}\hat{\alpha}_m\xi_{0,0}(k(\boldsymbol{r}-\boldsymbol{r}_m)+i\beta\boldsymbol{\eta}). \tag{22}$$

## IV. Numerical Experiments

To evaluate the effectiveness of the proposed method, we conducted numerical experiments. In the following experiments, a sound field generated by two point sources was observed using a microphone array, and we evaluated how effectively the proposed directional filtering could enhance the component originating from the target source in the estimated field. The speed of sound was set to $340.65\,\mathrm{m/s}$. First, we defined the mixed sound field $u_{\mathrm{mixed}}$ generated by two point sources $s_1$ (target) and $s_2$ (interference) as follows:

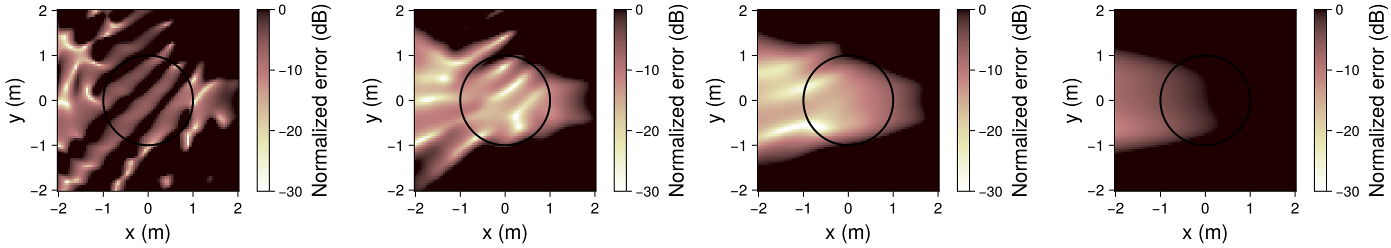
$$u_{\mathrm{mixed}}(\boldsymbol{r}) = A_1\frac{\exp(ik\|\boldsymbol{r}-\boldsymbol{r}_1\|)}{4\pi\|\boldsymbol{r}-\boldsymbol{r}_1\|}+A_2\frac{\exp(ik\|\boldsymbol{r}-\boldsymbol{r}_2\|)}{4\pi\|\boldsymbol{r}-\boldsymbol{r}_2\|}. \tag{23}$$

Here, $\boldsymbol{r}_1=[3.0,0.0,0.0]^{\mathsf{T}}\,\mathrm{m}$ and $\boldsymbol{r}_2=[0.0,3.0,0.0]^{\mathsf{T}}\,\mathrm{m}$ denote the positions of the point sources $s_1$ and $s_2$, respectively, and $A_1$ and $A_2$ are the complex amplitudes of each point source. In this experiment, both sources were set to have the same complex amplitude, i.e., $A_1=A_2$. Hereafter, we refer to the sound field generated by the target source $s_1$ as the target sound field, denoted by $u_{\mathrm{target}}$. Fig. 2 shows the mixed and target sound field at $400\,\mathrm{Hz}$. A hundred omnidirectional microphones were randomly placed inside a ball of radius $1\,\mathrm{m}$ centered at the origin, and the sound field was estimated using (16). The observation noise was generated as independent and identically distributed complex Gaussian noise such that the signal-to-noise ratio (SNR) was $20\,\mathrm{dB}$. The regularization parameter was set to $\lambda=0.01$. Then, the proposed directional




(a) (b) (c) (d)

Fig. 3: Real part of the estimated sound fields after applying the proposed directional filtering on the $xy$-plane ($z = 0$): (a) $\beta = 0$; (b) $\beta = 2$; (c) $\beta = 5$; (d) $\beta = 25$.




(a) (b) (c) (d)

Fig. 4: Normalized error distributions of the estimated sound fields after applying the proposed directional filtering on the $xy$-plane ($z = 0$): (a) $\beta = 0$; (b) $\beta = 2$; (c) $\beta = 5$; (d) $\beta = 25$.

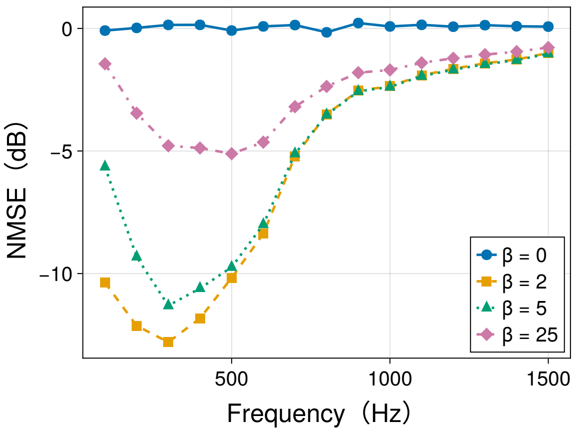



Fig. 5: NMSE versus frequency for different values of $\beta$.

filtering was applied to the estimated sound field using (22). The desired enhancement direction was set to be the direction from the origin to the target source $s_1$, i.e., $\boldsymbol{\eta} = [1.0, 0.0, 0.0]^\mathsf{T}$. As an evaluation metric, the normalized mean squared error (NMSE) was used, defined as

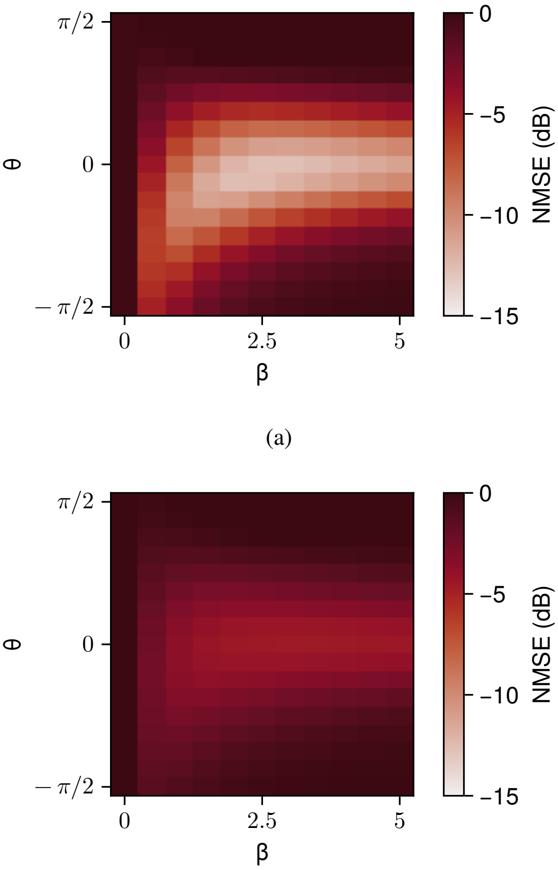
$$\mathrm{NMSE} := 10 \log_{10} \frac{\sum_{\mathrm{i} \in \mathrm{I}_{\mathrm{eval}}} |u_{\mathrm{target}}(\boldsymbol{r}_{\mathrm{eval}}^{(\mathrm{i})}) - (L_w u_{\mathrm{est}})(\boldsymbol{r}_{\mathrm{eval}}^{(\mathrm{i})})|^2}{\sum_{\mathrm{i} \in \mathrm{I}_{\mathrm{eval}}} |u_{\mathrm{target}}(\boldsymbol{r}_{\mathrm{eval}}^{(\mathrm{i})})|^2} (\mathrm{dB}). \tag{24}$$

Here, $u_{\mathrm{est}}$ is the estimated sound field. The set $\{\boldsymbol{r}_{\mathrm{eval}}^{(i)}\}_{i \in \mathrm{I}_{\mathrm{eval}}}$ represents the evaluation points, which are uniformly distributed on and inside a sphere of radius $1\,\mathrm{m}$ centered at the origin, at intervals of $0.05\,\mathrm{m}$. The black line in Figs. 2, 3, and 4 indicates the boundary of this evaluation region.

Fig. 3 shows the results for $\beta = 0$, 2, 5, and 25 at $400\,\mathrm{Hz}$. Note that $\beta = 0$ corresponds to the case where no directional filtering is applied, i.e., the estimated mixed sound field $u_{\mathrm{est}}$ itself. In addition, Fig. 4 presents the pointwise normalized error distributions between $L_w u_{\mathrm{est}}$ and $u_{\mathrm{target}}$ at $400\,\mathrm{Hz}$. These results confirm that the proposed directional filtering successfully enhances the sound field components in the desired direction $\boldsymbol{\eta}$.

Next, Fig. 5 shows the NMSE for different values of $\beta$ at various frequencies. It can be observed that the NMSE is lower in the low-frequency range, indicating that the resulting sound field is closer to the target sound field. In addition, excessively increasing $\beta$ tends to degrade performance, leading to higher NMSE values. This is likely due to the fact that $u_{\mathrm{target}}$ is a spherical wave, whereas $L_w u_{\mathrm{est}}$ increasingly approximates a plane wave as $\beta$ becomes large, resulting in a mismatch between the two.

Furthermore, we investigate the NMSE between $L_w u_{\mathrm{est}}$ and $u_{\mathrm{target}}$ with respect to variations in $\beta$ and $\boldsymbol{\eta}$. Here, we define $\boldsymbol{\eta} := [\cos\theta, \sin\theta, 0]^\mathsf{T}$ with $\theta \in [-\pi/2, \pi/2]$, and plotted the NMSE for varying $\beta$ and $\theta$ at $400\,\mathrm{Hz}$ and $800\,\mathrm{Hz}$ in Fig. 6. Even when $\boldsymbol{\eta}$ differs from the true direction of the target source, it is confirmed that, within an appropriate range of $\beta$, the proposed directional filtering improves the NMSE




(a) (b)

Fig. 6: NMSE of the estimated sound field after applying the proposed directional filtering with varying $\beta$ and $\boldsymbol{\eta}$: (a) 400 Hz; (b) 800 Hz.

compared to the case without any processing, i.e., $\beta = 0$.

## V. Conclusion

We proposed a sound field directional filtering method that enhances components arriving from a specified direction. Through numerical experiments, we quantitatively evaluated the effectiveness of the proposed directional filtering and confirmed its utility. In future work, we plan to explore the application of the proposed directional filtering to binaural rendering techniques.

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