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Implicit Interference Status Notification Through Time & Frequency Resource Selection in LoRaWAN

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Abstract—**Low-power wide area networks (LPWANs), such as long-range wide area network (LoRaWAN), have garnered significant attention for enabling long-range and low-power communication. Although end devices (EDs) can obtain local interference status information by detecting ongoing transmissions through carrier sense (CS), it's difficult for them to directly report such information to a gateway (GW) due to duty cycle (DC) restrictions. This paper proposes a method enabling each ED to notify GW of its local interference environment without additional signaling. The approach uses a portion of the payload bits to determine transmission timing and frequency channels. This allows the same bit sequence to map to different time-frequency resources based on the channels the ED avoids. By observing the received resource and recovering the bit sequence from the payload, the GW can then infer the pattern of avoided channels. Calculations confirm that this method achieves unique resource mapping and accurately avoids channel estimation without requiring protocol changes or signaling overhead.**

I. INTRODUCTION

The rapid growth of the Internet of Things (IoT) has driven the widespread use of wireless sensor networks (WSNs), composed of numerous compact, low-power sensors with wireless capabilities [1]. These networks often involve fixed deployment of many end devices (EDs) over wide areas, communicating intermittently at low data rates. To support battery-powered operation, EDs must be small, low-cost, and energy-efficient. In this context, low power wide area networks (LPWANs) have attracted attention for enabling long-range, low-power communication. Among them, long-range wide area network (LoRaWAN) is widely adopted for its scalability and ease of deployment [2], [3].

In LoRaWANs, EDs periodically transmit packets to the gateway (GW). LoRaWANs operate in the unlicensed band and are therefore subject to internal and external interference, resulting in packet collisions [4]. To ensure fairness in spectrum utilisation, LPWANs have duty cycle (DC) constraints, which limit the transmission time within a certain period. Carrier sense (CS), which checks the availability of the frequency channel to be used for a certain period of time immediately before packet transmission, is also mandatory.

To address these issues, packet-level index modulation (PLIM) was proposed as a means to improve throughput under DC constraints by embedding information in the timing and frequency of packet transmission [5]. In addition, a channel activity detection (CAD)-based PLIM was proposed [6] that allows an ED to back off and transmit when the CS detects another ED transmitting. This method detects other terminal transmissions at the CS, where the terminal checks the channel usage just before transmission, and backs off (postpones transmission). However, packet collisions occur between terminals in a relationship where they cannot detect each other's packet transmissions in the CS (hidden terminal problem). Based solely on the information obtained from the received packets, the radio environment information around the terminal is estimated, and radio resources are allocated based on this estimated information [7]. The impact of the hidden terminal problem is reduced without overhead by estimating the radio environment information on which terminals have backed off and which terminal's packet transmission was detected by the GW's CS, and using the obtained radio environment information for resource allocation. However, in this situation, considering the functional limitations of EDs and the aforementioned DC constraints, there is a need for a way for an interfering ED to indirectly communicate its local interference status to the GW without additional overhead in uplink (UL) communications.

Against this difficuty, this paper proposes an implicit interference status notification method. This method allows a device to implicitly communicate channels with a high level of interference (hereafter, avoided channels) to the GW by transmitting data packets without additional communication. It is capable of notifying the GW of up to two avoided channels. To achieve this, unlike conventional PLIM that primarily aims to increase data rates, the proposed method utilizes a portion of the payload bits as PLIM bits. An ED uses a unique one-to-one mapping rule between the PLIM bits and the avoided channels to determine its transmission resource (frequency channel and time slot). Since the same PLIM bit sequence maps to different resources depending on which channels are avoided, the GW, upon successfully receiving a packet and recovering the PLIM bits from its payload, can implicitly infer which channels were avoided.

This paper is organized as follows. Section II provides an overview of PLIM. Section III details the proposed resource mapping and interference inference mechanism. Section IV evaluates the effectiveness of the proposed scheme through numeric calculation. Finally, Section V concludes the paper.

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II. PACKET-LEVEL INDEX MODULATION (PLIM)

Without loss of generality, let us consider a scenario where a single ED communicates with a GW using K frequency channels. We assume that the ED generates data packets every T_{frame} [sec]. This time duration T_{frame} is defined as a frame length and is equally divided into Q time slots, each with a duration of T_{slot} [sec].

The ED generates an information bit sequence $B = (b_0, b_1, \ldots, b_{B-1})^\top$, where $b_i \in \{0, 1\}$ is the i-th information bit, and B is the length of the information bit sequence. Next, this information bit sequence B is divided into a PLIM bit sequence $B_{\text{plim}} = (b_0, b_1, \ldots, b_{B_{\text{plim}}-1})^\top$ and a payload bit sequence $B_{\text{pl}} = (b_{B_{\text{plim}}}, b_{B_{\text{plim}}+1}, \ldots, b_{B_{\text{plim}}+B_{\text{pl}}-1})^\top$. Here, B_{plim} and B_{pl} are the number of bits for the PLIM bit sequence and the payload bit sequence, respectively, such that $B_{\text{plim}} + B_{\text{pl}} = B$. Subsequently, the payload bit sequence B_{pl} is used to generate the main body of the data packet, following the same processing as conventional LoRaWAN. Finally, the PLIM bit sequence B_{plim} is used to transmit the data packet on a combination of frequency channel $k \in \mathcal{K} = \{0, 1, \ldots, K-1\}$ and time slot $q \in \mathcal{Q} = \{0, 1, \ldots, Q-1\}$, which is determined by a pre-shared index mapper $\mathcal{F}$ as shown in (1).

$$(k, q) = \mathcal{F}(B_{\text{plim}}). \tag{1}$$

The GW receives the data packet on frequency channel $\tilde{k} \in \mathcal{K}$ and estimates the received time slot $\tilde{q} \in \mathcal{Q}$ [5], [6]. The obtained pair of frequency channel and time slot $(\tilde{k}, \tilde{q})$ is input into the index demapper $\mathcal{F}^{-1}$, represented by (2), to demodulate the PLIM bit sequence $\tilde{B}_{\text{plim}} = (\tilde{b}_0, \tilde{b}_1, \ldots, \tilde{b}_{B_{\text{plim}}-1})^\top$.

$$\tilde{B}_{\text{plim}} = \mathcal{F}^{-1}(\tilde{k}, \tilde{q}). \tag{2}$$

Afterward, the payload of the data packet is demodulated using the same processing as conventional LoRaWAN to obtain the payload bit sequence $\tilde{B}_{\text{pl}} = (\tilde{b}_{B_{\text{plim}}}, \tilde{b}_{B_{\text{plim}}+1}, \ldots, \tilde{b}_{B-1})^\top$. Finally, the demodulated PLIM bit sequence $\tilde{B}_{\text{plim}}$ and the payload bit sequence $\tilde{B}_{\text{pl}}$ are combined to reconstruct the original information bit sequence $\tilde{B} = (\tilde{b}_0, \tilde{b}_1, \ldots, \tilde{b}_{B-1})^\top$.

III. PROPOSED METHOD: IMPLICIT INTERFERENCE NOTIFICATION METHOD

The overall system model is illustrated in Figure 1. Without loss of generality, we consider an UL transmission from an internal ED #B to the GW. The EDs in the figure are distinguished between internal system EDs (white icons: #A, #B, #C) and EDs from external system EDs (black icons: #X, #Y). Additionally, the red lightning bolt symbols indicate interference, with unfilled symbols representing interference 1 and filled symbols representing interference 2. We assume that Interference 1 and Interference 2 use different frequency channels. ED #B is shown to be affected by both Interference 1 from EDs #A and #X, and Interference 2 from EDs #C and #Y. In its UL transmission, ED #B avoids the channels used by both Interference 1 and Interference 2 and transmits data packets, thereby implicitly notifying the GW that it is being affected by Interference 1 and Interference 2. This system is capable of notifying the GW of up to two avoided channels.

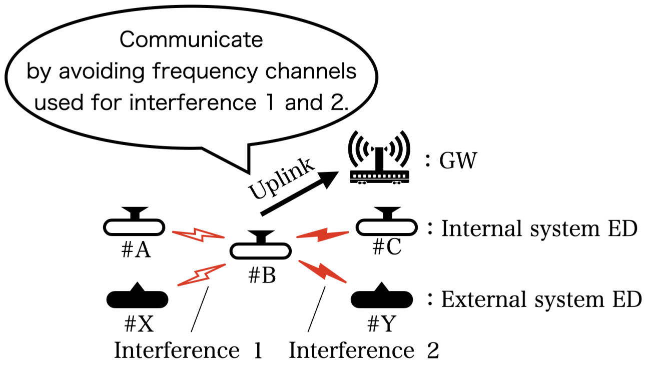

Fig. 1. System model

A. Resource Design Principles and Variable Definition

This method assigns a resource index (k, q) consisting of a frequency channel and a time slot, allowing an ED to avoid up to two channels. It is crucial to ensure that a given transmission code X maps to a unique (k, q), regardless of which channels are being avoided.

The set of all frequency channels is defined as $\mathcal{K} = \{0, 1, 2, \ldots, K-1\}$, where K denotes the total number of frequency channels. Similarly, the set of available time slots is defined as $\mathcal{Q} = \{0, 1, 2, \ldots, Q-1\}$, where Q is the total number of time slots. To ensure the uniqueness of the resource mapping, this method assumes that the system parameters satisfy $K \geq 4$ and $Q \geq K + 4$. The proof for these conditions will be provided in the Appendix.

To represent the availability of each frequency channel, we define a binary indicator a_k for each $k \in \mathcal{K}$, where $a_k = 1$ indicates that channel k is available, and $a_k = 0$ indicates that it is to be avoided.

$$a_k = \begin{cases} 1, & \text{if channel } k \text{ is available,} \\ 0, & \text{if channel } k \text{ is to be avoided.} \end{cases} \tag{3}$$

Using these indicators, the set of usable channels is defined as $\mathcal{K}_{\text{a}} = \{k \in \mathcal{K} \mid a_k = 1\}$, and the number of usable channels is denoted as $K_{\text{a}} = |\mathcal{K}_{\text{a}}|$. Accordingly, the total number of usable resources is $R = K_{\text{a}} \times Q$.

Let $\mathcal{K}_{\text{b}} = \{k \in \mathcal{K} \mid a_k = 0\}$ be the set of avoided channels. In this study, we assume that $|\mathcal{K}_{\text{b}}| \leq 2$, meaning at most two channels are avoided.

The transmission code X is derived from the PLIM bit sequence B_{plim}, device address B_{addr}, and packet counter B_{pcnt}. These bit sequences are converted into their decimal representations D_{plim}, D_{addr}, and D_{pcnt}, respectively. The transmission code X is then computed using these decimal representations as follows:

$$X = \bmod\left(D_{\text{plim}} + f(D_{\text{addr}}, D_{\text{pcnt}}), R\right). \tag{4}$$

Here, the modulo operation $\bmod(m, n)$ with $m \in \mathbb{N}$ and

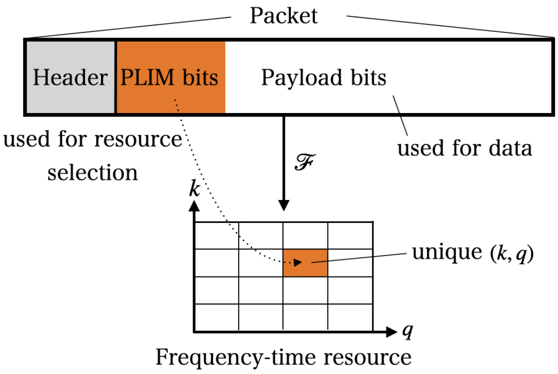

Fig. 2. Unique resource determination based on PLIM bits

$n \in \mathbb{N} \setminus \{0\}$ is defined as:

$$\mathrm{mod}\ (m, n) = m - \left(n \times \left\lfloor \frac{m}{n} \right\rfloor\right), \tag{5}$$

where $\lfloor \cdot \rfloor$ denotes the floor function. As defined in (5), $0 \leq \mathrm{mod}\ (m, n) < n$, and this definition is also applicable when m or n are negative values. $f(D_{\mathrm{addr}}, D_{\mathrm{pcnt}})$ is an arbitrary function uniquely determined by D_{addr} and D_{pcnt}.

B. Utilizing Payload Bits as PLIM Bits

In the context of the PLIM framework proposed in this paper, we use part of the payload as PLIM bits D_{plim}. These bits and resource indices enable ED to notify the GW of the interference environment. This design eliminates the need to transmit special signals or increase packet size.

Figure 2 illustrates this core concept: a portion of the payload bits is designated as a PLIM bit sequence. Header refers to the conventional LoRaWAN PHY header and is described as a configuration that includes the MAC header and preamble. This method maintains compatibility with the LoRaWAN frame structure while enabling implicit interference notification.

Even with the same value for the transmission code X, the actual combination of frequency channel and time slot used for transmission changes based on which channels the ED avoids. This one-to-one mapping means the receiver can accurately deduce which channels the transmitter avoided simply by observing the received resource index (k, q) and the PLIM bits recovered from the payload. The proofs are discussed in the Appendix. By intelligently embedding interference-related information into the resource mapping through existing payload bits, our approach enables effective communication without any additional overhead or changes to the packet structure.

C. Mapping to Resource Index (k, q)

The mapping from the transmission code X to the final resource index (k, q) begins with an initial offset that depends on the avoided channels.

The initial frequency channel offset k_{start} is defined as:

$$k_{\mathrm{start}} = \begin{cases} \mathrm{mod}\ \left(\sum_{k \in \mathcal{K}_{\mathrm{b}}} k + 1,\ K\right), & \text{if } \mathcal{K}_{\mathrm{b}} \neq \emptyset, \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

The initial time slot offset q_{start} is determined by the indices of the avoided channels. If the two avoided channels are denoted as b_0 and b_1 with $b_0 < b_1$, then:

$$q_{\mathrm{start}} = \begin{cases} b_0 + 1, & \text{if } \mathcal{K}_{\mathrm{b}} = \{b_0, b_1\},\ b_0 < b_1, \\ 1, & \text{if } \mathcal{K}_{\mathrm{b}} = \{K - 1\}, \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

Next, we compute the effective index $k_{\mathrm{start}}^{\mathrm{idx}}$ among the usable channels corresponding to k_{start}. This $k_{\mathrm{start}}^{\mathrm{idx}}$ represents the count of available channels up to the k_{start}-th channel within the set of usable channels $\mathcal{K}_{\mathrm{a}}$.

$$k_{\mathrm{start}}^{\mathrm{idx}} = \sum_{k=0}^{k_{\mathrm{start}}-1} a_k. \tag{8}$$

Using $k_{\mathrm{start}}^{\mathrm{idx}}$, we determine the usable-channel-based frequency channel index k_{idx} corresponding to X:

$$k_{\mathrm{idx}} = \quad \mathrm{mod}\ (k_{\mathrm{start}}^{\mathrm{idx}} + X,\ K_{\mathrm{a}}). \tag{9}$$

The actual frequency channel resource index k is then selected as the one that satisfies the following condition. This condition explicitly states that k is the (k_{idx})-th available channel within $\mathcal{K}_{\mathrm{a}}$. This ensures that the mapping skips over any avoided channels while assigning channel indices continuously among the usable ones.

$$k = \left\{ i \;\middle|\; a_i = 1 \text{ and } \sum_{j=0}^{i-1} a_j = k_{\mathrm{idx}} \right\}. \tag{10}$$

Finally, the time slot resource index q is computed using:

$$q = \quad \mathrm{mod}\ \left(q_{\mathrm{start}} + \left\lfloor \frac{k_{\mathrm{start}}^{\mathrm{idx}} + X}{K_{\mathrm{a}}} \right\rfloor,\ Q\right). \tag{11}$$

This formula ensures that the time slots increase cyclically after incrementing through all available frequency channels. Of particular importance is the fact that different combinations of avoided channels have different starting points in the resource grid. As a result, even if the same transmission code X is used, the resource index (k, q) finally allocated will always be unique, guaranteeing a one-to-one mapping for all avoidance patterns.

D. Estimation of Avoided Channel Patterns

In this PLIM framework, it is essential not only to uniquely determine the transmission resource index (k, q) from the transmission code X, but also to enable the receiver to infer which channels were intentionally avoided during mapping. This capability is made possible precisely because the proposed mapper ensures a one-to-one correspondence between the transmission code and the resource index (k, q), even under various channel avoidance conditions.

At the receiver, transmission code X is assumed to be embedded in the packet. The receiver can obtain both the received resource index (k, q) and the PLIM bits. Since the mapping is uniquely designed for every possible avoided channel, the receiver can perform reverse mapping from $(X,(k, q))$ to identify the avoided channel pattern used by the transmitter.

An avoided channel pattern is represented as binary data $\{a_k\}_{k=0}^{K-1}$, where $a_k = 0$ indicates that channel k is prohibited and $a_k = 1$ indicates that it is available. The receiver performs an exhaustive search over all valid patterns under the assumption that at most two channels are avoided. The total number of candidate avoided channel patterns depends on the number of prohibited channels $|\mathcal{K}_\mathrm{b}|$ and is given by the binomial coefficient:

$$P_{|\mathcal{K}_\mathrm{b}|} = \binom{K}{|\mathcal{K}_\mathrm{b}|} = {}_K C_{|\mathcal{K}_\mathrm{b}|} = \frac{K!}{(K - |\mathcal{K}_\mathrm{b}|)!\,|\mathcal{K}_\mathrm{b}|!},$$

where $\binom{n}{k}$ (or ${}_n C_k$) denotes the number of possible combinations of k elements selected from n elements.

Accordingly, the total number of candidate patterns for $|\mathcal{K}_\mathrm{b}| = 0, 1, 2$ is:

$$\sum_{|\mathcal{K}_\mathrm{b}|=0}^{2} \binom{K}{|\mathcal{K}_\mathrm{b}|} = 1 + K + \binom{K}{2},$$

which corresponds to the number of patterns for avoiding 0, 1, and 2 channels, respectively.

For each candidate pattern $\{a_k\}$, the receiver computes the hypothetical location of the resource index (k', q') using the same mapping logic as the transmitter. This involves deriving the channel index k from the effective index k_idx, using the channel selection condition described in (10). The final resource index (k, q) is then obtained by applying a pattern-specific initial offset and a linear scan of the available resource space.

If the calculated (k', q') matches the received (k, q), then the avoided channel pattern tested is considered the one used by the transmitter. Due to the one-to-one nature of the mapping, only one pattern will satisfy this condition for any given (k, q), allowing the receiver to identify the avoided channels reliably.

Thus, this brute-force detection scheme, although exhaustive, remains computationally feasible due to the limited number of candidate patterns. More importantly, it leverages the structure of the proposed notification-oriented mapper, which guarantees that even with the same transmission code X, the resulting (k, q) will differ across avoided channels. This unique separability across patterns forms the foundation for avoided channel estimation, which will be theoretically validated in the Appendix.

IV. NUMERIC CALCULATION RESULTS

This section evaluates the proposed implicit notification method. The focus is placed on how to avoid channels affecting the final mapped resource (k, q), and whether the receiver can infer the pattern correctly.

TABLE I
NUMERIC CALCULATION PARAMETERS

Parameter	Value
Total number of frequency channels, K	4
Total number of time slots, Q	8
Number of avoided channels, $\lvert\mathcal{K}_\mathrm{b}\rvert$	0, 1, 2
PLIM bits, D_plim	1,3
Packet counter, D_pcnt	0
Device address, D_addr	0

TABLE II
MAPPING AND ESTIMATED AVOID CHANNEL FOR $X = 1, 3$

$X = D_\mathrm{plim}$	$\mathcal{K}_\mathrm{a}$	(k, q)	$\hat{\mathcal{K}}_\mathrm{a}$
1	$[1,1,1,1]$	$(1,0)$	$[1,1,1,1]$
	$[1,0,1,1]$	$(3,0)$	$[1,0,1,1]$
	$[1,1,0,0]$	$(1,4)$	$[1,1,0,0]$
3	$[1,1,1,1]$	$(3,0)$	$[1,1,1,1]$
	$[1,0,1,1]$	$(2,1)$	$[1,0,1,1]$
	$[1,1,0,0]$	$(1,3)$	$[1,1,0,0]$

A. Numeric Calculation Parameters

Table I summarizes the main parameters used in our calculations. Table 1 summarizes the main parameters used in the numerical calculations. The PLIM bit value depends on the number of avoided channels. 4 bits for no avoided channels, 3 bits for one avoided channel, and 2 bits for two avoided channels. Focus on the transmission code X with packet counter D_pcnt and device address D_addr fixed to 0 and only D_plim varied. Thus, $X = D_\mathrm{plim}$ is defined.

B. Examples of Resource Mapping

Table II presents calculation results that illustrate how the transmission resource (k, q) varies according to the usable channels $\mathcal{K}_\mathrm{a}$. $\mathcal{K}_\mathrm{a}$ is expressed in binary in (3). Since it is difficult to enumerate all channel situations, three situations are shown without loss of generality: [1,1,1,1] where all channels are available, [1,0,1,1] where channel 1 is avoided, and [1,1,0,0] where channels 2 and 3 are avoided. In this calculation, the transmitted code X depends solely on the PLIM bit value and is therefore defined as $X = D_\mathrm{plim}$. The examples shown for $X = 1$ and $X = 3$ demonstrate that, even for the same PLIM value, the resulting resource changes depending on the avoided channel. Since X is mapped to different resources based on the specific configuration of avoided channels, the gateway can infer the local interference environment of the transmitter by comparing the received resource (k, q) with the PLIM bits extracted from the payload. This confirms that even with a fixed X, the resulting resource changes depending on the avoid channel. Consequently, implicit interference status notification is achieved without any additional signaling.

V. CONCLUSION

This paper proposed a method for implicit interference status notification in wireless communication systems. This method utilizes a portion of the transmitted payload bits as PLIM bits that determine the resource index on which the packet is to be transmitted. A key advantage of our approach is that it enables the transmitter to notify the receiver of up to two intentionally avoided channels without incurring additional control signaling overhead, as PLIM bits are generated from the existing payload. This is achieved by employing a one-to-one mapping scheme where the mapping between the PLIM bit sequence and the selected resource index, i.e., a combination of frequency channel and time slot, is varied based on the channels the transmitter aims to avoid. This means that the same PLIM bits lead to different resource selections when combined with different avoided channel patterns. Consequently, upon successful packet reception, the receiver can precisely infer the interference status at the specific ED, specifically which channels were avoided by the EN, simply by observing the received resource index and decoding the PLIM bits from the payload.

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APPENDIX

A. Proof 1 (Requirements: $K \geq 4$ and $Q \geq K+4$)

Given the total number of frequency channels K and time slots Q, this proof shows that for any combination of PLIM bits D_{plim} and avoid channel $\mathcal{K}_{\mathrm{b}}$, the resulting resource (k, q) is uniquely determined. The proof proceeds by contradiction. In other words, we prove that the following propositions hold.

$$\mathcal{F}(D_{\mathrm{plim}}, \mathcal{K}_{\mathrm{b},1}) \neq \mathcal{F}(D_{\mathrm{plim}}, \mathcal{K}_{\mathrm{b},2}).$$

Assume the proposition is false. That is, assume there exist two different avoid channels $\mathcal{K}_{\mathrm{b},1} \neq \mathcal{K}_{\mathrm{b},2}$ such that, for the same PLIM bits D_{plim}, the resulting resource (k, q) is identical $\mathcal{F}(D_{\mathrm{plim}}, \mathcal{K}_{\mathrm{b},1}) = \mathcal{F}(D_{\mathrm{plim}}, \mathcal{K}_{\mathrm{b},2})$.

In the proposed mapping scheme, the number of usable frequency channels K_{a} differs depending on the avoid channel. For example, if one channel is avoided, $K_{\mathrm{a}} = K - 1$; if two channels are avoided, $K_{\mathrm{a}} = K-2$. Thus, even if D_{plim} is fixed, the resulting resource index (k, q) varies because the frequency index cycles with different periodicities across different avoid channels.

The frequency channel index k is calculated as (10), and the time slot index q is determined by (11), where k_{start} and q_{start} are offset values uniquely determined by the avoid channel $\mathcal{K}_{\mathrm{b}}$.

Because K_{a}, k_{start}, and q_{start} are all dependent on $\mathcal{K}_{\mathrm{b}}$, they differ between $\mathcal{K}_{\mathrm{b},1}$ and $\mathcal{K}_{\mathrm{b},2}$. As a result, even for the same D_{plim}, both k and q will differ.

When $K < 4$, wrap-around due to the modulo operation may result in (k, q) collisions. However, under the assumption that $K \geq 4$ and $Q \geq K + 4$, the diversity of resource configurations and the differing offsets prevent such collisions.

Thus, the initial assumption leads to a contradiction. Therefore, the proposition holds for any given D_{plim} and avoids channel $\mathcal{K}_{\mathrm{b}}$; the resulting resource (k, q) is uniquely determined.

B. Proof 2 (Uniqueness of Resource Mapping)

Given the total number of frequency channels K and time slots Q, we show that for any combination of PLIM bits D_{plim} and avoid channel $\mathcal{K}_{\mathrm{b}}$, the resulting resource (k, q) is uniquely determined. The proof is conducted by contradiction. In other words, we prove that the following propositions hold.

$$\mathcal{F}(D_{\mathrm{plim}}, \mathcal{K}_{\mathrm{b},1}) = \mathcal{F}(D_{\mathrm{plim}}, \mathcal{K}_{\mathrm{b},2}) = (k, q).$$

We assume that the device address $D_{\mathrm{addr}} = 0$ and the packet counter $D_{\mathrm{pcnt}} = 0$ so that the transmission code becomes $X = D_{\mathrm{plim}}$. This simplification follows directly from (4), which defines X as a function of D_{plim}, D_{addr}, and D_{pcnt}.

Assume the proposition is false. That is, suppose there exist two different avoid channels $\mathcal{K}_{\mathrm{b},1} \neq \mathcal{K}_{\mathrm{b},2}$ such that, for the same D_{plim}, the same resource is assigned $\mathcal{F}(D_{\mathrm{plim}}, \mathcal{K}_{\mathrm{b},1}) = \mathcal{F}(D_{\mathrm{plim}}, \mathcal{K}_{\mathrm{b},2}) = (k, q)$.

In the proposed mapping scheme, the number of usable frequency channels K_{a} is determined as $K_{\mathrm{a}} = K - |\mathcal{K}_{\mathrm{b}}|$, and the values of k_{start} and q_{start} are uniquely determined by the avoid channel $\mathcal{K}_{\mathrm{b}}$. Accordingly, for each pattern, the resource is determined by (10) (11).

Because $\mathcal{K}_{\mathrm{b},1} \neq \mathcal{K}_{\mathrm{b},2}$, it follows that either K_{a}, k_{start}, or q_{start} must differ between the two patterns. This means that the modulo periodicity (influencing k) and the slot offset behavior (influencing q) differ. Hence, even with the same D_{plim}, the resulting (k, q) pair will differ unless a rare wrap-around alignment occurs.

However, under the assumptions $K \geq 4$ and $Q \geq K + 4$, the range of valid (k, q) pairs is sufficiently large and diverse to prevent such alignment across differing patterns.

This contradicts the original assumption that (k, q) is the same for different avoid channels. Therefore, by contradiction, the proposition holds for any given D_{plim} and the set of avoid channels $\mathcal{K}_{\mathrm{b}}$, the resulting (k, q) is uniquely determined.

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