Robust Superdirective Beamforming Using a Uniform Circular Array with Directional Microphones

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***Abstract*—Superdirective beamformers with uniform circular arrays are attractive in practical applications since they exhibit high directivity factors (DFs) and steering flexibility. High DF is desired for acoustic noise and reverberation suppression. However, this beamformer is very sensitive to array imperfections due to its low white noise gains (WNGs) at low frequencies of speech. Recently, a robust superdirective beamformer with a closed-form generalized sidelobe canceller (GSC) structure has been proposed to achieve a tradeoff between the WNG and the DF via a rank-reduced blocking matrix of the GSC. In this paper, we propose a method to boost the robust superdirective beamformer based on two improvements: Firstly, we extend the robust superdirective beamformer to uniform circular arrays and leverage directional microphones to enhance the performance of superdirective beamformers; Secondly, we propose to design the blocking matrix via a particular differential beamforming method, i.e., circular differential directional microphone arrays. A comparative study shows that the proposed method brings about a noticeable improvement in WNG at the low frequencies (15 to 20 dB at 1 kHz) over the existing method, meanwhile enhancing the DF and exhibiting a more frequency-invariant beampattern.**

Index Terms: microphone array, directional microphone, superdirective beamforming, circular array, differential beamforming

I. Introduction

Superdirective beamformers [1] have attracted considerable attention [2]–[6] due to their ability to provide high directivity factors (DFs) with small array apertures. However, these beamformers are extremely sensitive to array imperfections, such as sensor noise, mismatches between sensor responses, and position errors, especially at the low frequencies of speech [7], which limits their practical use for voice capture. The sensitivity to these imperfections can be evaluated by the white noise gain (WNG), which is a helpful measure of the robustness of the beamformer [8]–[11]. Superdirective beamformers with controlled WNG are designed by incorporating a WNG constraint into the optimization problem [1], [12]–[15]. These optimizations, such as convex optimization [15], lack control over the directivity and shape of beampatterns.

Recent work [16] proposes a robust superdirective beamformer (RSD) via controlling the number of nulls in the beampattern to achieve a tradeoff between the WNG and DF. Essentially, the robust superdirective beamformer utilizes a closed-form generalized sidelobe canceller (GSC) structure as the solution and controls the dimension of the blocking matrix in the GSC to determine the number of nulls for its directivity patterns. Therefore, the blocking matrix is vital to the final performance of the robust superdirective beamformer. In [16], the blocking matrix is obtained via the Gram-Schmidt process by iteratively finding an orthonormal basis for the null space of the steering vector at the target direction. The robust superdirective beamformer is designed for a uniform linear array (ULA) with omnidirectional microphones.

In this work, we propose a design method for the robust superdirective beamformer, hereafter referred to as RSD+, featuring two main improvements: 1) We extend the robust superdirective beamformer to a uniform circular array (UCA) equipped with directional microphones. Inspired by recent advances demonstrating that directional microphones can significantly improve WNG in differential beamforming [17]–[20], we leverage these microphones to enhance the performance of superdirective beamformers. Meanwhile, this extension subsumes the conventional UCA with omnidirectional microphones as a special case. 2) We propose designing the blocking matrix using a particular differential beamforming method, specifically circular differential directional microphone arrays (CDDMA) [17]. CDDMA not only ensures the null space property required for effective blocking but also improves robustness in spatial filtering. Through analyses and simulations, both improvements from RSD+ result in an increased WNG at low frequencies while enhancing the DF and exhibiting a more frequency-invariant beampattern.

II. Problem Formulation

We consider a UCA consisting of M first-order directional microphones with a radius of r. Without loss of generality, we assume all microphones lie in the x-y plane. In the following, we assume that the directional microphones in the array have a fixed elevation angle of $\frac{\pi}{2}$. As depicted in Fig. 1, the directional

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979-8-3315-7206-8/25/$31.00

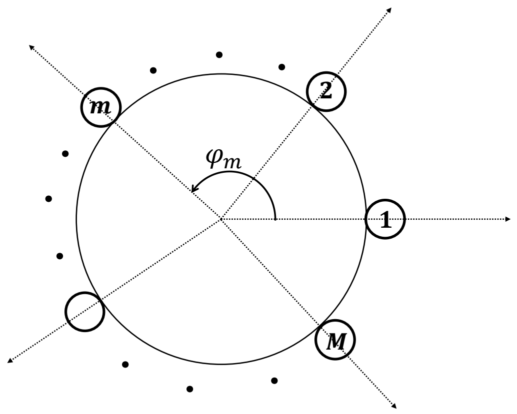

Fig. 1. Uniform circular array with M directional microphones, where φ_m denotes the azimuth angle of the mth directional microphone, and the arrow points in the direction of the main lobe of the directional microphone.

microphones are evenly distributed on the circle and pointed outward along the diameter, and the angular position of the mth microphone is

$$\varphi_m = \frac{2\pi(m-1)}{M}. \tag{1}$$

We assume that a plane wave impinges on the array with an incident direction of azimuth angle θ and elevation angle ϕ. In this scenario, the steering vector is given by

$$\mathbf{d}\left(\omega, \theta, \phi, p\right) = \left[d_1, \cdots, d_m, \cdots, d_M\right]^T, \tag{2}$$

where the superscript $(\cdot)^T$ is the transpose operator and each element in (2) can be obtained as

$$\begin{aligned} d_m &= e^{j\frac{\omega r}{c}\cos(\theta-\varphi_m)\sin\phi} \\ &\times [p + (1-p)\cos(\theta-\varphi_m)\sin\phi], \end{aligned} \tag{3}$$

where $j = \sqrt{-1}$ is the imaginary unit, $\omega = 2\pi f$ is the angular frequency, f is the temporal frequency, p defines the spatial characteristic of the directional microphone. For instance, different values of p make the well-known patterns: 1) cardioid for $p = 0.5$; 2) omnidirectional for $p = 1$; 3) dipole for $p = 0$. Thus, the conventional UCA with omnidirectional microphones is a special case of the proposed UCA with directional microphones.

A superdirective beamformer is classified as a fixed beamformer, functioning as a time-invariant, data-independent spatial filter. A fixed beamformer $\mathbf{h}(\omega)$ is designed for a desired steering direction $(\theta_{\text{d}}, \phi_{\text{d}})$, where θ_{d} is the desired azimuth angle, and ϕ_{d} is the desired elevation angle. The beamformer $\mathbf{h}(\omega)$ exhibits a distortionless response in the desired steering direction. In contrast, in undesired directions, the beamformer demonstrates a certain level of suppression, such that

$$\mathbf{d}^H(\omega, \theta, \phi, p)\mathbf{h}(\omega) \begin{cases} = 1, & \text{if } \theta = \theta_{\text{d}} \text{ and } \phi = \phi_{\text{d}}, \\ < 1, & \text{otherwise}, \end{cases} \tag{4}$$

where the superscript $(\cdot)^H$ is the conjugate-transpose operator. In the following, we omit the ϕ_{d} since we assume desired sources are from x-y plane with $\phi_{\text{d}} \triangleq \frac{\pi}{2}$.

For the sake of completeness, we briefly introduce three widely used performance measures for fixed beamformers, namely WNG, beampattern, and DF. WNG is the most convenient way to evaluate the robustness of a beamformer to some of its imperfections, such as sensor noise, position errors, etc [10]. Assuming a distortionless response in the desired steering direction, the WNG is defined as

$$\mathcal{W}[\mathbf{h}(\omega)] = \frac{1}{\mathbf{h}^H(\omega)\mathbf{h}(\omega)}. \tag{5}$$

Beampattern illustrates the directional sensitivity of a beamformer to a plane wave impinging on the array from the incident angle θ, ϕ:

$$\mathcal{B}[\mathbf{h}(\omega), \theta, \phi] = \mathbf{d}^H\left(\omega, \theta, \phi, p\right)\mathbf{h}(\omega). \tag{6}$$

The DF is defined as the ratio between the signal power of the array output in the desired steering direction and the power averaged over all directions [8]:

$$\mathcal{DF}[\mathbf{h}(\omega)] = \left\{\frac{1}{4\pi}\int_0^{\pi}\int_0^{2\pi} \sin\phi \left|\mathcal{B}[\mathbf{h}(\omega), \theta, \phi]\right|^2 \mathrm{d}\theta \ \mathrm{d}\phi\right\}^{-1}. \tag{7}$$

In the following, we omit the dependency on ω to simplify the presentation. Naturally, all notations are assumed to be calculated independently for each narrowband frequency.

The superdirective beamformer is obtained to maximize DF. The optimization problem is equivalent to

$$\min \quad \mathbf{h}^H\mathbf{\Gamma}\mathbf{h} \quad \text{s.t.} \quad \mathbf{d}^H(\theta_{\text{d}}, p)\mathbf{h} = 1, \tag{8}$$

where $\mathbf{\Gamma}$ is the spherically isotropic noise field matrix for the proposed UCA with directional microphones and is defined as

$$\mathbf{\Gamma} = \frac{1}{4\pi}\int_0^{\pi}\int_0^{2\pi} \sin\phi \ \mathbf{d}\left(\theta, \phi, p\right)\mathbf{d}^H\left(\theta, \phi, p\right)\mathrm{d}\theta \ \mathrm{d}\phi. \tag{9}$$

The superdirective beamformer for the proposed array is

$$\mathbf{h}_{\text{SD}} = \frac{\mathbf{\Gamma}^{-1}\mathbf{d}(\theta_{\text{d}}, p)}{\mathbf{d}^H(\theta_{\text{d}}, p)\mathbf{\Gamma}^{-1}\mathbf{d}(\theta_{\text{d}}, p)}. \tag{10}$$

To improve the robustness, a WNG constraint could be added to the optimization, then the matrix $\mathbf{\Gamma}$ can be replaced by a regularized form as $\mathbf{\Gamma} + \epsilon\,\mathbf{I}_M$, where $\epsilon \geq 0$ is a parameter to control the amount of diagonal loading and facilitates a compromise between the DF and WNG for the beamformer [21]. However, obtaining the optimal value for this parameter can be challenging. Moreover, its impact on the pattern of the final superdirective beamformer is difficult to control.

III. Proposed Beamformer

In this paper, we propose a method based on an alternative approach for designing a robust superdirective beamformer first described in [16]. This approach allows for a trade-off between WNG and DF by controlling the number of nulls in the final pattern. The method described in [16] is intended for linear arrays and omnidirectional microphones. The present work extends this method to a uniform circular array with directional microphones.

Firstly, [16] offers a GSC-like solution for superdirective beamforming:

$$\mathbf{h}_{\text{SD}} = \mathbf{h}_{\text{DS}} - \mathbf{B}(\mathbf{B}^H \mathbf{\Gamma} \mathbf{B})^{-1} \mathbf{B}^H \mathbf{\Gamma} \mathbf{h}_{\text{DS}}, \tag{11}$$

where $\mathbf{B}$ is a blocking matrix with size $M \times (M-1)$, and $\mathbf{h}_{\text{DS}}$ is the delay and sum beamformer, obtained by:

$$\mathbf{h}_{\text{DS}} = \frac{\mathbf{d}(\theta_{\text{d}}, p)}{\mathbf{d}^H(\theta_{\text{d}}, p)\mathbf{d}(\theta_{\text{d}}, p)}. \tag{12}$$

For the problem considered in this work, only the matrix $\mathbf{B}$ in (11) is unknown. In principle, the matrix $\mathbf{B}$ functions as a blocking matrix, with each column vector being orthogonal to the steering vector in the desired direction. To obtain the matrix $\mathbf{B}$, [16] proposed a method with three steps: In the first step, a Gram-Schmidt orthonormalization process is used to obtain an initial matrix $\mathbf{B}_{\text{init}}$ of size $M \times (M-1)$. The second step calculates the quantities:

$$\zeta_i = \mathbf{b}_i^H \mathbf{\Gamma} \mathbf{b}_i \text{ for } i \in \{1, 2, \ldots, M-1\} \tag{13}$$

where $\mathbf{b}_i$ is the i-th column of $\mathbf{B}_{\text{init}}$. Therefore, each column $\mathbf{b}_i$ corresponds to one quantity ζ_i. This quantity measures the output-to-input power ratio of the diffuse noise (i.e., the inverse of the directivity factor) for $\mathbf{b}_i$. In the third step, column vectors in $\mathbf{B}_{\text{init}}$ are reordered to form the final matrix $\mathbf{B}$, i.e.,

$$\mathbf{B} = [\mathbf{b}_1, \cdots, \mathbf{b}_i, \cdots, \mathbf{b}_{M-1}], \tag{14}$$

where corresponding ζ_i calculated by i-th column $\mathbf{b}_i$ becomes larger with increasing i, i.e., $\zeta_1 \leq \cdots \leq \zeta_i, \leq \cdots \leq \zeta_{M-1}$. It should be noted that the GSC-like solution in (11) is equivalent to the traditional solution in (10) and has not enhanced robustness [22].

Secondly, to improve the robustness, a so-called robust superdirective beamformer [16] is given by

$$\mathbf{h}_{\text{RSD}} = \mathbf{h}_{\text{DS}} - \mathbf{B}_J(\mathbf{B}_J^H \mathbf{\Gamma} \mathbf{B}_J)^{-1} \mathbf{B}_J^H \mathbf{\Gamma} \mathbf{h}_{\text{DS}}, \tag{15}$$

where $\mathbf{B}_J$ with size $M \times J$ ($J \leq M-1$) is obtained by selecting the first J column vectors of $\mathbf{B}$ as follows

$$\mathbf{B}_J = [\mathbf{b}_1, \cdots, \mathbf{b}_j, \cdots, \mathbf{b}_J], \tag{16}$$

where J determines the number of nulls in the beampattern, and a higher J would result in a higher-order beampattern, thereby influencing the maximum directivity [16]. This relationship demonstrates a trade-off between DF and WNG for the robust superdirective beamformer.

The study conducted in [16] utilizes the Gram-Schmidt process to find an orthonormal basis for the null space of $\mathbf{d}^H(\theta_{\text{d}}, \phi_{\text{d}}, p)$ such that $\mathbf{d}^H(\theta_{\text{d}}, \phi_{\text{d}}, p)\mathbf{B}_{\text{init}} = \mathbf{0}_{M-1}$. However, $\mathbf{B}_{\text{init}}$ returned by the Gram-Schmidt process is not unique. Meanwhile, the way $\mathbf{B}_J$ is selected from $\mathbf{B}_{\text{init}}$ is also not unique [16]. The current ordering results in the selection of blocking filters corresponding to the small quantity ζ_i. Since this quantity measures the inverse of the directivity factor, the small quantity ζ_i implicitly implies a small WNG. Therefore, this ordering might also lead to using blocking filters with a low WNG - this would not be ideal.

TABLE I
DMA DIRECTIVITY PATTERN SPECIFICATIONS.

Condition (J)	Coefficients $(\{\theta_1, \ldots, \theta_{\text{j}}, \ldots, \theta_{\text{J}}\})$
2	$\{\theta_{\text{d}} + \frac{7}{9}\pi, \theta_{\text{d}} - \frac{7}{9}\pi\}$
4	$\{\theta_{\text{d}} + \frac{1}{2}\pi, \theta_{\text{d}} - \frac{1}{2}\pi, \theta_{\text{d}} + \frac{7}{9}\pi, \theta_{\text{d}} - \frac{7}{9}\pi\}$
6	$\{\theta_{\text{d}} + \frac{1}{2}\pi, \theta_{\text{d}} - \frac{1}{2}\pi, \theta_{\text{d}} + \frac{2}{3}\pi, \theta_{\text{d}} - \frac{2}{3}\pi, \theta_{\text{d}} + \frac{8}{9}\pi, \theta_{\text{d}} - \frac{8}{9}\pi\}$

In addition to using cardioid microphones, we propose using the CDDMA differential beamformer [17] to design a new blocking matrix $\mathbf{W}_J$, where $J \leq M-1$. Here, we replace $\mathbf{B}_J$ by

$$\mathbf{W}_J = [\mathbf{w}_1, \cdots, \mathbf{w}_j, \cdots, \mathbf{w}_J], \tag{17}$$

where $\mathbf{w}_j^H$ is a first-order CDDMA differential beamformer whose spatial null points to the desired direction of the superdirective beamformer. CDDMA is a specific method used to design a maximized WNG differential beamformer for UCA with directional microphones [17]. Thus, $\mathbf{W}_J$ can fulfill the null space property. We formulate a linear system of equations as below to obtain the first-order differential beamformer $\mathbf{w}_j^H$:

$$\mathbf{R}_j \mathbf{w}_j = [1, 0]^T, \tag{18}$$

where the constraint matrix $\mathbf{R}_j$ of size $2 \times M$ is given by

$$\mathbf{R}_j = \begin{bmatrix} \mathbf{d}^H(\theta_{\text{j}}, p) \\ \mathbf{d}^H(\theta_{\text{d}}, p) \end{bmatrix}, \tag{19}$$

where θ_{j} is an empirically defined angle as the desired direction of the differential beamformer. We employ the minimum-norm solution, which maximizes the WNG and thereby enhances the robustness of the beamformer, to solve our linear system equations as shown in (18). Each column $\mathbf{w}_j$ in $\mathbf{W}_J$ is obtained by [17]:

$$\mathbf{w}_j = \mathbf{R}_j^H [\mathbf{R}_j \mathbf{R}_j^H]^{-1} [1, 0]^T. \tag{20}$$

In our proposed method, we only design J different differential beamformers, and the column ordering in $\mathbf{W}_J$ can be arbitrary. Thus, the proposed robust superdirective beamformer using CDDMA is given by

$$\mathbf{h}_{\text{RSD+}} = \mathbf{h}_{\text{DS}} - \mathbf{W}_J(\mathbf{W}_J^H \mathbf{\Gamma} \mathbf{W}_J)^{-1} \mathbf{W}_J^H \mathbf{\Gamma} \mathbf{h}_{\text{DS}}. \tag{21}$$

IV. Simulations and Analysis

In this section, we examine the performance of the proposed RSD+ in terms of the beampattern, WNG, and DF. We compare this method to the conventional RSD calculated in (15). We consider UCAs consisting of $M = 7$ microphones with a radius of $r = 2$ cm, and we design all superdirective beamformers with a desired direction at $\theta_{\text{d}} = \pi$. To design our proposed superdirective beamformer $\mathbf{h}_{\text{RSD+}}$ in (21), we set θ_{j} using the values in Table I.

In Fig. 2, we study the beampattern of the RSD+ in 2D via setting $\phi = \frac{\pi}{2}$ in (6). We compare the beampatterns of RSD+ using cardioid microphones ($\mathbf{h}_{\text{RSD+}}$ with $p = 0.5$) with those of RSD using cardioid microphones ($\mathbf{h}_{\text{RSD}}$ with $p = 0.5$) and RSD using omnidirectional microphones ($\mathbf{h}_{\text{RSD}}$ with $p = 1$) for

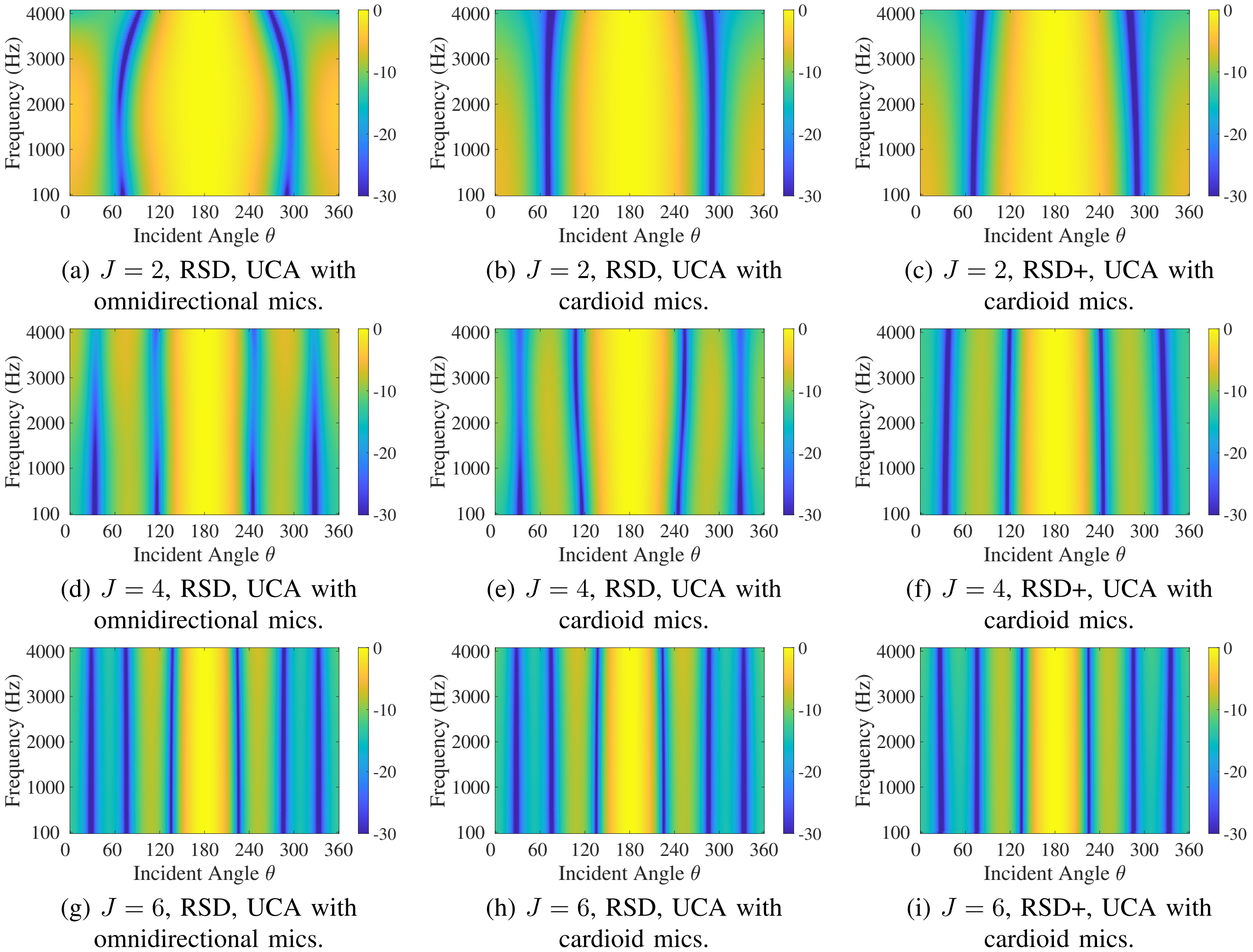

Fig. 2. Beampattern analysis of RSD+. The beampatterns corresponding to (a),(d), and (g) result from the conventional RSD using omnidirectional microphones. The beampatterns corresponding to (b),(e), and (h) result from the conventional RSD using cardioid microphones. The beampatterns corresponding to (c),(f), and (i) result from the proposed RSD+ using cardioid microphones.

different conditions $J \in \{2, 4, 6\}$. For the conditions $J = 2$ and $J = 4$, only cardioid microphones can produce beampatterns of RSD with deeper nulls. The proposed RSD+ exhibits more frequency-invariant patterns and a narrower mainlobe at high frequencies than the RSD under these conditions. For the condition $J = 6$, the beampatterns of all the methods are identical. Meanwhile, we can see that J determines the number of nulls in a superdirective beamformer's beampattern, which is related to the order of the beampattern. The maximum order of a superdirective beamformer for a UCA is $\lfloor \frac{M}{2} \rfloor$ [23], which is different from a superdirective beamformer for a ULA, where the maximum order is $M - 1$ [16]. Therefore, when $J = 6$, all methods achieve the maximum order shown in Fig. 2, rendering all robust superdirective beamformers equivalent to the traditional superdirective beamformer as calculated using (10).

Figure 3 compares the RSD+ to the RSD in terms of WNG and DF for various conditions $J \in \{2, 4, 6\}$. For the WNG corresponding to the condition $J = 2$ in Fig. 3(a), we observe that methods using cardioid microphones can achieve a frequency-invariant WNG that is significantly higher than that of the method using omnidirectional microphones. Since $J = 2$ corresponds to the first-order pattern, the cardioid microphones inherently produce first-order patterns themselves. Consequently, the algorithms primarily adjust the null positions, potentially resulting in a frequency-invariant WNG with comparable performance for both RSD and RSD+. For the WNG corresponding to $J = 4$ in Fig. 3(c), we observe that the cardioid microphones can boost the WNG of RSD by approximately 10 dB at 1 kHz compared to omnidirectional microphones. Furthermore, the proposed RSD+ can improve the WNG at 1 kHz by an additional 10 dB. Consequently, the proposed RSD+ using cardioid microphones can improve the overall WNG at 1 kHz by 20 dB compared to the RSD using omnidirectional microphones. For the DF with $J = 2$ and $J = 4$ depicted in Figs. 3(b) and 3(d), the proposed RSD+ using cardioid microphones consistently achieves the highest DF. However, we can see that cardioid microphones cannot continuously improve the DF for the RSD. For the condition $J = 6$, all the methods reach the identical maximum DF. However, the method using cardioid microphones still demonstrates 15 dB WNG improvement at 1 kHz compared

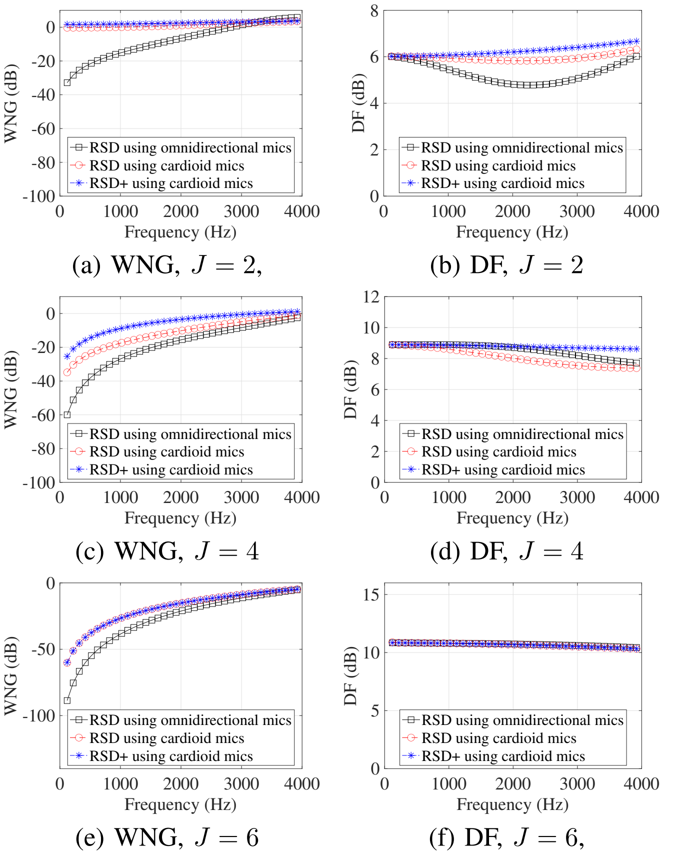

Fig. 3. WNG and DF comparison among RSD using omnidirectional microphones, RSD using cardioid microphones, and RSD+ using cardioid microphones.

to the method using omnidirectional microphones. In summary, the cardioid microphones can continually improve the superdirective beamformer's WNG. Moreover, the proposed RSD+ with cardioid microphones can consistently achieve the best WNG and DF for different conditions J. Meanwhile, the condition J introduces a tradeoff between WNG and DF for superdirective beamformers, where a higher J results in a higher DF but a lower WNG, and vice versa.

V. Conclusions

This paper proposes a robust superdirective beamformer design using differential beamforming and directional microphones for a uniform circular array, referred to as RSD+. The proposed RSD+ achieves a more frequency-invariant pattern and improves the low-frequency WNG by 15 to 20 dB at 1 kHz without sacrificing DF, compared to the conventional RSD using omnidirectional microphones. Additionally, we examine the effect of directional microphones on the performance of RSD in terms of WNG and DF. While directional microphones can always improve the WNG, they may sometimes do so at the expense of the DF.

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