# A Comparison Study of GRAPPA and Generalized Series Methods for parallel MRI at high acceleration factor

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***Abstract*—A comparison study of GRAPPA and Generalized Series for parallel MRI images is performed and analyzed at high acceleration factor. Through extensive simulation experiments on various data sets, the conventional GRAPPA method proved its efficiency at low acceleration factor when acquiring few auto-calibrating lines. However, at high acceleration factor 8-12 with very sparse $k-$space sampling, the GS method proved to be more superior by significantly reducing noise level and achieving much higher accuracy. The key to success of GS method is sufficient amount of low frequency info contained in reference image. This can be achieved by collecting a sufficient the number of ACS lines. In this study, 15%-20% of total $k-$space lines was sufficient.**

***Index Terms*—pMRI, undersample, GRAPPA, GS, reconstruction method, high acceleration factor**

## I. INTRODUCTION

Magnetic Resonance Imaging (MRI) is an advanced imaging technology where images are generated based on the magnetic resonance phenomenon of the nucleus in the tissue. One of main challenges in MRI is long acquisition time. Patients, especially children, have difficultly staying still or holding breath during this time. As result, image has motion artifacts. One way to address long acquisition time is deploying parallel MRI (pMRI). Instead of using a large homogeneous volume receiver coil, in pMRI multiple phase-arrayed RF coils [1] are used to acquire reduced $k-$space data at once. Signal from each individual RF coil covers only a portion of the object being imaged [2]. However, acquiring partial data violates Nyquist sampling rate, therefore causing aliasing artifact in the Fourier reconstructed image. In order to combat with under-sampling artifacts, advanced image reconstruction methods such as SENSE (Sensitivity Encoding) [3] and GRAPPA (GeneRalized Autocalibrating Partially Parallel Acquistions) [4] have been developed for pMRI and used commonly used on clinical scanners [5], [6]. SENSE technique requires additional coil sensitivity information to eliminate the effect of under-sampling the k-space, while GRAPPA seeks to regenerate the missing data in the k-space domain [7]. GRAPPA is an efficient method for pMRI image reconstruction, however it still yields a significant amount of aliasing artifact and residual noise when data is highly-undersampled [8]. In this work, we consider the problem of pMRI image reconstruction from a different perspective by utilizing a Generalized Series (GS) model [9]. The method uses a reference image and represents missing dynamic frequency components using the set of GS basis functions. A comparison study of GRAPPA and GS reconstruction methods in the case of highly under-sampled data acquisition scheme is performed. Results show that the GS method gained a higher reduction of both under-sampling artifacts and residual noise level in case of high acceleration factor.

## II. MATERIALS AND METHODS

### *A. Review of GRAPPA reconstruction method*

GRAPPA is an effective method for image recovery when parallel receivers are used. In GRAPPA, $k-$space data are initially undersampled in the phase-encoding direction $k_y$ in order to accelerate the MRI scan time. The acceleration factor $R$ is defined as the ratio of the amount of $k$-space data required for a fully sampled image to the amount of $k-$space data collected in an accelerated acquisition. Reducing amount of $k-$space lines violates the Nyquist criterion and shrinks the field of view (FOV) [6], resulting in an aliased image. The idea behind GRAPPA is to use portions of the acquired k-space to calculate the portion that were not acquired. GRAPPA is based on the fact that any given k-space point can be predicted from its neighboring $k$-space points. Therefore, a missing data point can be reconstructed by combining neighboring points together in the appropriate way. The neighborhood of acquired data used for extrapolating a missing data point is defined as a GRAPPA kernel. Specifically, the missing phase-encoding $k$-space line at coil $j$ and position $m\Delta k_y$ is estimated as follows:

$$\mathbf{D}_j(k_y - m\Delta k_y) = \sum_{l=1}^{L}\sum_{b=0}^{N_b} w(j,b,l,m)\mathbf{D}_l(k_y - bR\Delta k_y) \quad (1)$$

where $\mathbf{D}$ is the $k-$space data matrix, $N_b$ is number of the kernel used in the reconstruction, $w(j, b, l, m)$ represents the weights used to expand the linear combination; $L$ is the number of coils, $\Delta k_y$ represents the sampling interval

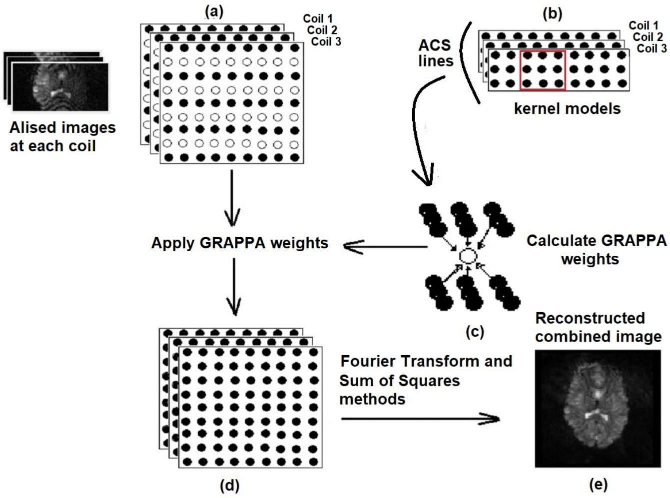


Fig. 1. Illustration to the GRAPPA reconstruction method.[10]

along phase-encoding direction and $m = 1,..,R-1$ . A schematic diagram of the GRAPPA algorithm is shown in Fig. 1. To use GRAPPA, the weights in combination of the acquired points must be found to recover missing data. An additionally acquired portion of k-space known as the auto-calibration signal (ACS) is applied to determine GRAPPA weights according (2) as follows:

$$\widehat{\mathbf{w}} = (\mathbf{D}^T\mathbf{D})^{-1}\mathbf{D}^T\mathbf{D}_{ACS}, \tag{2}$$

where $D_{ACS}$ is additional acquisition matrix. Therefore, GRAPPA is known as an autocalibrated method. ACS lines are usually chosen at the center area of $k-$space. The minimum number of ACS lines required for GRAPPA is $R-1$. After estimating weights $\widehat{\mathbf{w}}$, the missing $k-$space data of each coil is calculated using Eq. (1). Uncombined single coil images $I_l(\mathbf{r})$ are obtained by taking the Fourier Transform and then combined using a conventional sum of squares (SOS) method [11] as follows:

$$I(\mathbf{r}) = \sqrt{\sum_{l}^{L} |I_l(\mathbf{r})|^2}. \tag{3}$$

## B. Generalized Series Model

The generalized series (GS) model was originally developed by Liang et al. to address constrained image reconstruction problems [9]. In this paper we apply a modified GS reconstruction method from Nguyen et al. [12] to pMRI data. Specifically, a MRI image at a certain coil is modelled as

$$\rho(\boldsymbol{r}) = I_{\text{ref}}(\boldsymbol{r}) \sum_{l=-L/2}^{L/2-1} c_l e^{-j2\pi l \Delta \boldsymbol{k}\cdot\boldsymbol{r}}, \tag{4}$$

where $I_{\text{ref}}(\boldsymbol{r})$ is the high-resolution reference which serves as a prior information for reconstruction, $c_l$ is the GS coefficients and $l \in \left\{-\frac{L}{2}, -\frac{L}{2}+1, \ldots, \frac{L}{2}-1\right\}$. $I_{ref}$ is suggested to be chosen such that it contains low frequency features. Therefore, in this study, we obtain reference image from the center ACS lines in order to preserve basic smooth features of the object being imaged.

To achieve the reconstructed image $\rho$, knowledge about GS coefficients $c_l$ is a requirement. This value has been estimated by minimizing the following total-variation regularized cost function

$$\widehat{\boldsymbol{c}} = \arg\min_{c} \|\boldsymbol{d} - \boldsymbol{S}\boldsymbol{\Psi}\boldsymbol{c}\|_2^2 + \lambda\|\boldsymbol{c}\|_2^2, \tag{5}$$

where $d$ is the measured $k$-space data that should combine low frequency area and high frequency area [9], [12], $S$ is the sampling and measurement operator, $\Psi$ is the matrix whose columns represents the GS basic functions derived from (4), and $\lambda$ denotes the regularization parameter [9]. The choice of GS basis functions $\Psi$ depends on the characteristics of the time-varying signal component that is missing in the data with respect to the reference image. In this paper, sampling GS space covering both low and high frequency region is suggested. The above optimization problem is solved using the Conjugate Gradient optimization method. Having estimated $\widehat{c}$, image at single coil can be estimated using (4).

# III. Experimental Results

## A. Simulation study

Experimental data used in this study is the MRI data of knee provided by [13] with 15 signal coils and 32 slices. The data were acquired using SIEMENS 3T MRI scanner with echo time TE = 22 ms, repetition time TR = 2800 ms, $150^0$ flip angle and 770x768 in-plane image resolution. Image in plane resolution is 770x768 sensor coil. GRAPPA and GS reconstruction method was performed at the acceleration factor of 8 and using 169 ACS lines. Fig. 2 shows the resulting under-sampling k-space trajectory and ACS lines. A comparison study between GRAPPA and GS reconstruction methods is performed at high acceleration factor in terms of efficiency in reducing aliasing artifacts and noise level. With the same measured data including both acquired data and ACS $k-$space lines, we got the the Cartesian trajectory for input simulation scenario as Fig. 2

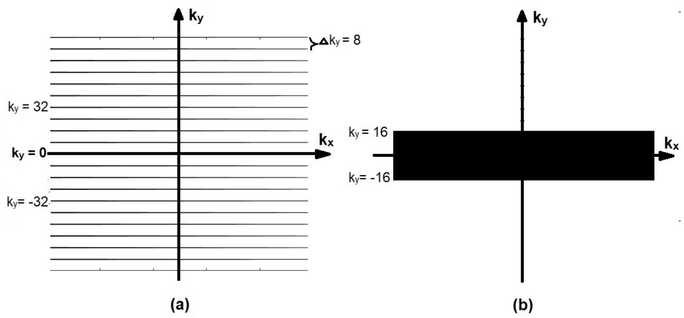


Fig. 2. (a) Under-sampled k-space trajectory; (b) Additional ACS lines.

In order to quantitatively evaluate the accurateness of GRAPPA and GS method, the error map $\mathbf{E}$ and the Normalized

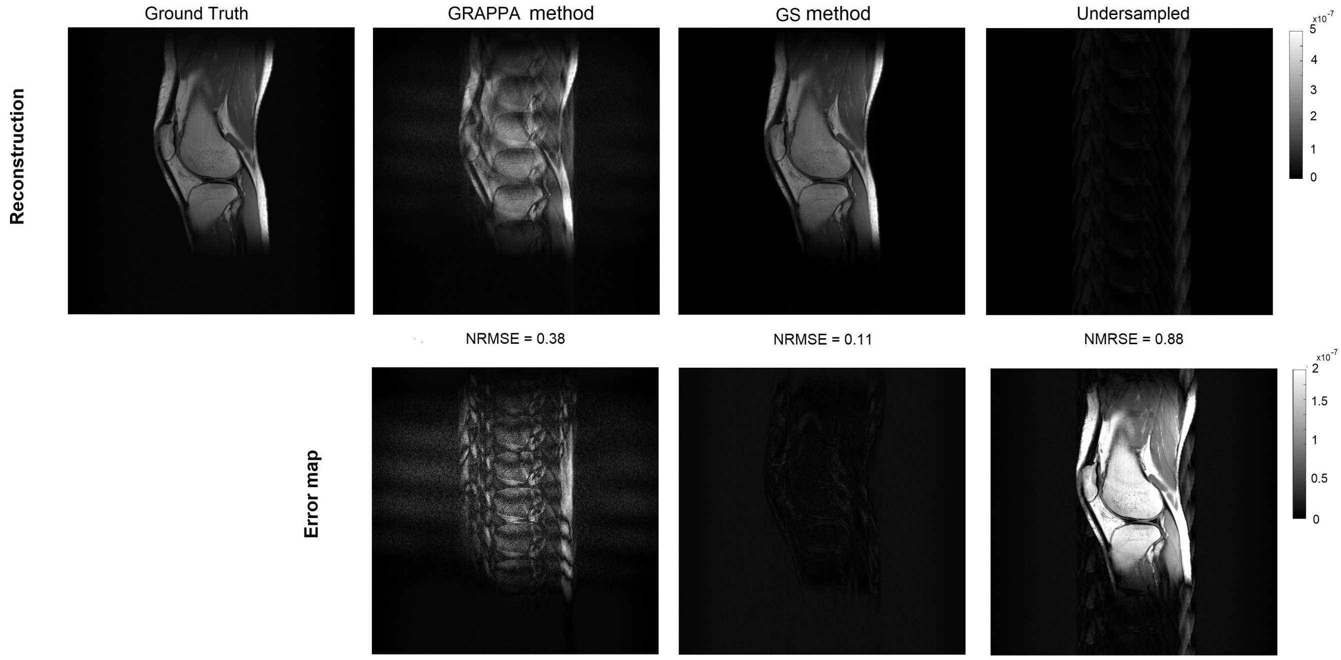


Fig. 3. Reconstructed images of GRAPPA and GS method when comparing to Ground Truth and under-sampled images at acceleration factor R = 8, 169 ACS lines

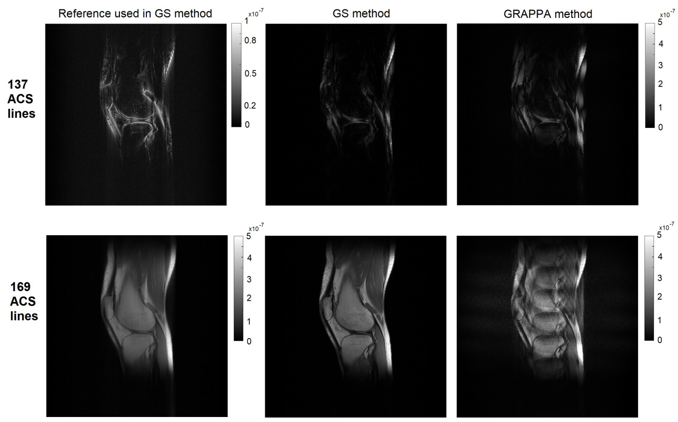


Fig. 4. Effect of the number of ACS lines and role of reference image for GRAPPA and GS reconstruction methods at acceleration factor $R = 8$

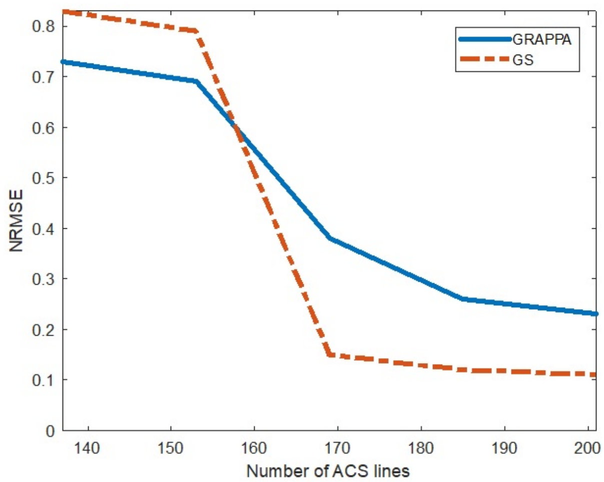


Fig. 5. NRMSEs of GRAPPA and GS reconstruction methods as a function of the number of ACS lines

Root Mean Square Error (NRMSE)was used as follows:

$$\begin{aligned}\mathbf{E}(\widehat{I}) &= |\widehat{I} - I|, \\ \mathrm{NRMSE}(\hat{I}) &= \frac{\|\widehat{I} - I\|_2}{\|I\|_2}.\end{aligned} \tag{6}$$

Noise in MRI can seriously affect the accuracy of the measurements. Signal-to-Noise ratio (SNR) [14] is a key parameter to determine the efficiency of any given imaging experiment, characterizing the degree to which noise affects the quality of an MR image. Conventionally, SNR can be calculated as follows:

$$\mathrm{SNR}(dB) = 20 \log_{10} \left( \frac{\mathrm{Mean}(|\mathrm{Signal}|)}{\mathrm{Mean}(|\mathrm{Noise}|)} \right) \tag{7}$$

where signal and noise is defined as the magnitude value at voxels in the chosen signal and noise region of interest respectively. A region of interest showing the signal and a region of interest representing the noise is selected to calculate the mean value of the signal and the standard deviation of the noise respectively. In this experiment, a major region inside

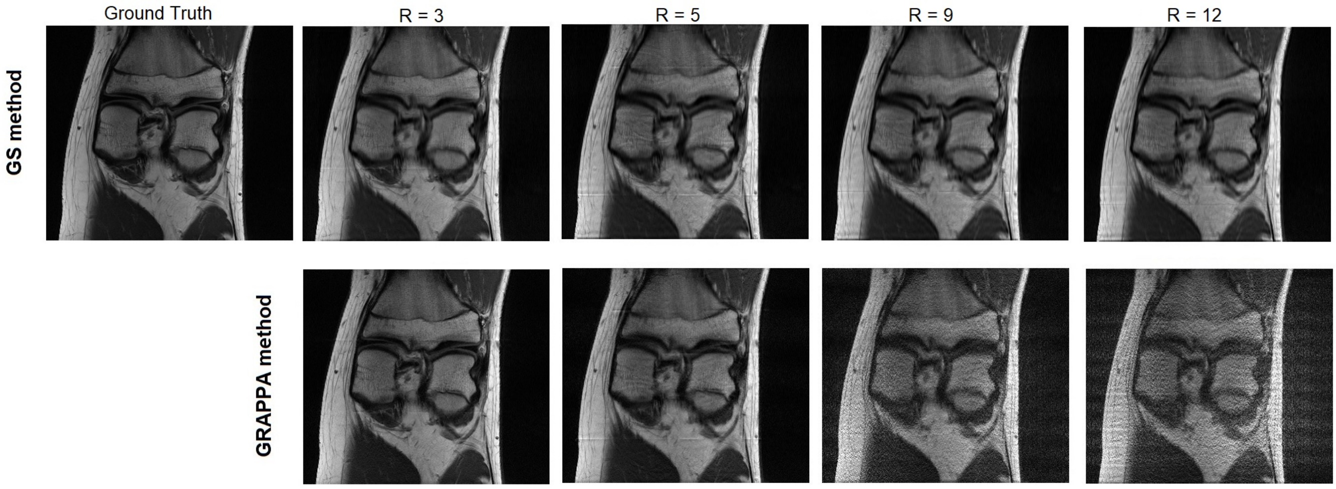


Fig. 6. Reconstructed images using GRAPPA and GS methods when altering acceleration factor from R = 3 to R = 12

of the knee area is chosen as the signal, while region outside is chosen as the noise. Fig. 3 shows the reconstructed images obtained from GRAPPA and GS method at $R = 8$ experiment and 169 ACS lines. It can be clearly to seen that the GRAPPA method successfully reconstructs image from undersamped data but it still yields residual some amount of aliasing and significant amount of noise. On the other hand, GS methods reduces aliasing effects significantly and the corresponding error map does not show obvious structures. NRMSE quantity also shows the significant difference between the GRAPPA and GS methods when NRMSE of GS reconstruction is much smaller than that of GRAPPA method.

### B. Effect of the number of ACS lines

The following experiment is simulated to assess the role of ACS lines used in both GRAPPA and GS methods. The number of ACS lines was varied from 137 to 201 lines while acceleration factor $R$ was fixed at 8. Fig. 5 shows the NMRSEs of GRAPPA and GS methods. As mentioned in [12], reference image for GS is recommended to contain sufficient low frequency information which on this work is suggested to be extracted from the center ACS lines. When this demand is met and sufficient the number of ACS lines is used, the GS method yielded superior result compared to GRAPPA method. This is clearly demonstrated in Fig. 4 which shows GRAPPA and GS reconstruction using 137 and 169 ACS lines. We found that collecting 15% to 20% on of the total number of phased-encoding lines was sufficient for GS method to give results better than GRAPPA method. On the other hand, decreasing the number of ACS lines below some threshold leads to a poor quality of images reconstructed by GS method. The NRMSE plot as a function of the number of ACS lines confirmed the observed finding.

### C. Effect of acceleration factor

Experiments in sections A and B proved that the efficiency of GS method was better than that of GRAPPA at the acceleration factor $R = 8$. We further investigate the performance

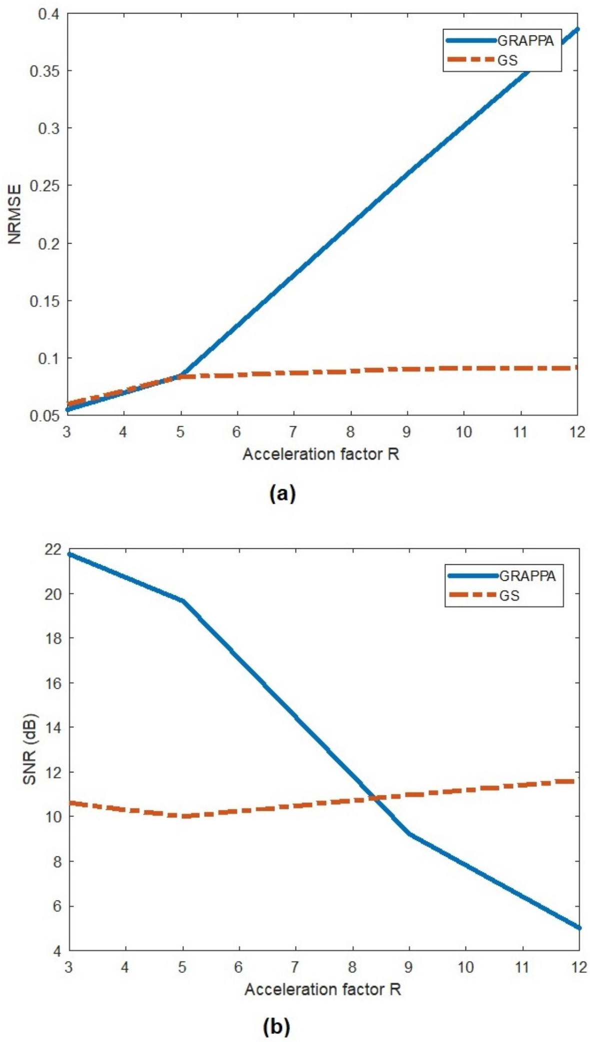


Fig. 7. (a) NMRSE and (b) SNR of GRAPPA and GS reconstruction method as a function of acceleration factor.

of both methods when $R$ is varied. On a different knee data set with 320x368 in-planed resolution, 38 slices and 15 coils [13]. Data was collected from SIEMENS 3T scanner, with TE = 50ms, TR = 4300ms and $180^0$ flip angle. Fig. 6 shows the reconstructed images when altering acceleration factor $R$ and keeping the number of ACS lines to be fixed at 46 lines. It can be seen that the quality of GRAPPA image obviously decreases at high acceleration factor with noticeable residual amount of noise. Unlike GRAPPA, quality of GS images only changes slightly. At large rate $R = 12$, the GS method clearly gave higher accuracy.

Relationship between acceleration factor R and SNR as well as NRMSE was studied in Fig. 7. The graph shows that NRMSE of GRAPPA rises significantly from 0.05 to 0.4 while its SNR quantity reduces from 22dB to 4dB as acceleration factor increases from $R = 3$ to $R = 12$. On the other hand, reconstructed images from GS method has NRMSE increasing slightly from 0.05 to 0.08 and SNR oscillating around 11dB. At a smaller factor $R = 3$, GRAPPA method yielded reconstructed image with higher resolution than that of GS method. However, in general the GRAPPA method prove less efficient than the GS method at large acceleration factors, with noticeably significant residual aliasing and noise.

### D. *Effect on noise data*

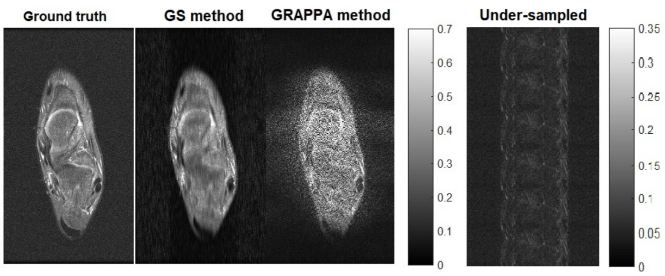


Fig. 8. Reconstruction images of GRAPPA and GS on noisy data

In order to evaluate the noise sensitivity of GRAPPA and GS reconstruction methods, a noisy anatomical pMRI data set was used with TE = 60ms, TR = 4500ms, $111^0$ flip angle and 8 channel coils. Fig. 8 shows that the reconstructed images using GRAPPA and GS methods at acceleration factor of 6 and using 31 ACS lines. It can be clearly seen that the GS method reduced noise significantly while GRAPPA reduced under-sampling aliasing at expense of significant of noise.

## IV. Conclusions

Parallel MRI contributes valuable achievements in fast MRI area through conventional image reconstruction methods such as SENSE and GRAPPA. In this paper, we considered pMRI image reconstruction from a different perspective by utilizing a modified GS method. We further analyzed and compare GRAPPA method with GS method. It was found that GRAPPA proved its efficiency to recover images at low acceleration factor by reducing aliasing effects. However, at high acceleration factor, the GS method outperforms the GRAPPA method. The success of GS method significantly depends on the amount of low frequency information contained in reference image. Furthermore, GS also significantly reduces noise level. The GS reconstruction method reveals its advantage for pMRI at high acceleration factor and with images containing low-frequency features.

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