FDA Radar Ambiguity Function Optimization With Simulated Annealing Algorithm

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***Abstract*—Different from phased-array providing range-independent beampattern, frequency diverse array (FDA) yields range-dependent transmit beampattern. Since ambiguity function is an effective tool to analyze the time-delay (range) and Doppler resolution performance of various radar systems, this paper derives the FDA radar ambiguity function and optimizes the ambiguity function with simulated annealing algorithm, with an aim to focus the transmit energy to the desired range-angle section and suppress undesired sidelobes. The proposed method is verified by numerical results.**

***Index Terms*—Frequency diverse array, radar ambiguity function, FDA radar, simulated annealing algorithm.**

I. Introduction

Although phased-array is employed for various applications, its beam steering is fixed in an angle for all ranges and thus, the range and angle of targets cannot be directly estimated from its beamforming output due to an inherent range ambiguity. To overcome this disadvantage, frequency diverse array (FDA) was proposed [1]–[3]. The most important FDA difference from conventional phased-arrays is that a small frequency increment, compared to the carrier frequency, is applied across the elements. The frequency increment results in that the beam direction changes as a function of the range, angle and time. It can be considered as a specific transmit beamforming [4], [5] applied on phased-array. Nevertheless, FDA is significantly different from conventional phased-array which beam direction is independent of the range and time in far-field.

Due to its promising application potentials, FDA has sparked much investigations. FDA was investigated in [6], [7], [27] as a range-dependent beam with applications in suppressing range ambiguous clutter. Additional work was reported in [9] to exploit the benefit of applying the FDA for synthetic aperture radar (SAR) high-resolution imaging. Sammartino and Baker [10] investigated frequency diverse MIMO radar which employs nonlinear element spacings. Higgins and Blunt [11] explored range and angle coupled beamforming with frequency diverse chirp signals. It is also possible for range and angle localization of targets [12], [13]. However, current researches concentrate mainly on analyzing FDA range-dependent beampattern characteristics [14].

Nevertheless, how to optimally design FDA parameters is still an open question. Since FDA transmit beampattern can be controlled by tuning the frequency increments, we proposed transmit subaperturing FDA radar for range and angle estimation [12] and derived the FDA radar Cramér-Rao lower bounds (CRLBs) for estimating direction, range and velocity [15]. Optimal frequency diverse subarray design with CRLB minimization was discussed in [16]. However, due to the range and angle dependent FDA beampattern and multiple targets have different range and angle positions, the CRLB to multiple targets cannot be uniformly minimized and they are minimized separately.

Since directly optimizing FDA parameters is a non-convex problem and difficult to solve, this paper attempts to design FDA parameters with ambiguity function optimization. Ambiguity function [17] is a useful tool for evaluating performance metrics of various radars such as delay and Doppler resolutions and the probabilities of detection and false alarm [18]–[23]. It also helps in selecting appropriate system parameters depending on the required performance criteria. For these reasons, Chen and Vaidyanathan [24] used the MIMO radar ambiguity function [25] to design frequency-hopping waveforms. This work is further extended by Khan et al. [26] for phased-MIMO radar with colocated antennas.

In this paper, we derive FDA radar ambiguity function and then optimally design the FDA parameters via ambiguity function optimization. The algorithm aims to focus the transmit energy to desired range-angle section and suppress undesired sidelobes. The remaining sections are organized as follows: Section 2 derives the FDA ambiguity function in a general way. Section 3 optimizes the ambiguity function with the simulated annealing algorithm. Section 4 provides numerical results. Finally, Section 5 concludes the paper.

II. FDA Radar Ambiguity Function

At a first glance FDA technique resembles frequency scanning technique accomplished in conventional antenna arrays. In frequency scanning technique, the antenna elements are supposed to receive the same (frequency-wise) but delayed waveform and it is the delay responsible for the beam scanning. However, in FDA technique the frequency is scanned continually and each antenna element in the array radiates distinct time points of the waveform.

Moreover, different from a phased-array, FDA can be excited by either the same waveform [27] or different waveforms [28]. But, unlike conventional multiple-input multiple-output

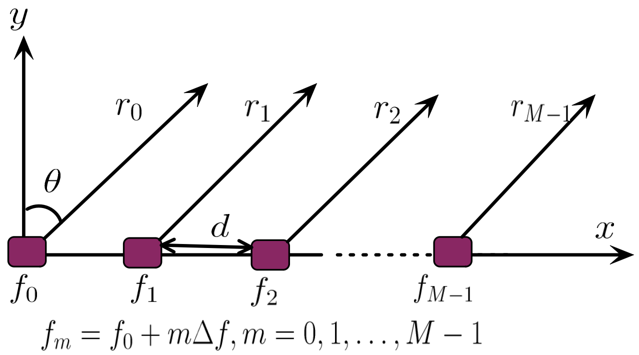

Fig. 1. Illustration of a linear FDA transmitter.

(MIMO) radar using orthogonal waveforms [29], FDA should be coherent waveforms across the elements to provide transmit gain. For simplicity and without loss of generality, we assume first that the narrow-band monochromatic signal radiated from each element is identical but with a frequency increment, Δf, as shown in Figure 1.

Suppose the mth element transmitted signal is

$$\phi_m(t) = e^{j2\pi q_{m,1} t} s(t) \quad (1)$$

where $q_{m,1}$ being the mth transmit signal pulse carrier frequency and $s(t)$ is

$$s(t) \triangleq \begin{cases} 1, t \in [0, T_\phi] \\ 0, \text{otherwise} \end{cases},$$

with T_ϕ being the pulse duration. The mth FDA element emits the signal $\phi_m(t)$ with a frequency increment Δf_{m}. The signals emitted by the mth element can be modeled as

$$\begin{aligned} s_m(t) &= \sqrt{M}\phi_m(t) e^{-j2\pi f_m t} w_m^* \\ &= \sqrt{M} u_m(t) w_m^* \end{aligned} \quad (2)$$

where

$$f_m = f_0 + \Delta f_{\mathrm{m}} \quad (3)$$

with f_0 being the FDA radar carrier frequency, is the radiation frequency, w_m is weights, * is the conjugate operator and $u_m(t)$ is

$$u_m(t) = e^{-j2\pi(f_0 - q_{m,1} + \Delta f_{\mathrm{m}})t} s(t). \quad (4)$$

The $\sqrt{M}$ is used to obtain an identical transmit power constraint.

Throughout this paper, we assume a narrowband system where the propagation delays manifest as phase shifts to the transmitted signals. Following [12], the transmitted signal propagating in the azimuth angle θ and range r can be represented by

$$S_{\mathrm{m}}(t, \theta, r) = \sqrt{M} \sum_{\mathrm{m}=1}^{M} w_m^* a_m(t, \theta, r) u_m(t) \quad (5)$$

where $a_m(t, \theta, r)$ can be approximately written as [30]

$$a_m(t, \theta, r) = \exp\left\{ j2\pi \left(\Delta f_m t - \frac{\Delta f_m r}{c_0} + m\frac{d f_0 \sin\theta}{c_0} \right) \right\} \quad (6)$$

For simplicity and without loss of generality, we use a single-antenna in the receiver. Suppose the round-trip time delay to a target in the angle θ and range r is τ and $w_m = 1$ is used. The received signal can then be expressed as

$$y(t, \tau, \upsilon, \theta, r) = \sqrt{M} \sum_{m=1}^{M} a_m(t, \theta, r) \times u_m(t - \tau) e^{j2\pi \upsilon t} \quad (7)$$

where υ is the Doppler frequency. When the received signal is matched with the assumed parameters $(\tau', \upsilon', \theta', r')$, the matched filter output can then be defined as the general FDA radar ambiguity function (8), where

$$\chi_{m,m'}(\tau, \upsilon) \triangleq \int_0^{T_\phi} u_m(t) u_{m'}^*(t + \tau) e^{j2\pi \upsilon t} dt \quad (9)$$

III. Optimization of the Frequency Increment Code of FDA Radar

We propose an optimization algorithm to choose for the frequency increment code which optimizes the FDA radar ambiguity function. This is done by imposing a cost function which puts penalties on these peak values. This forces the energy of the ambiguity function $\chi(\tau, \upsilon, \theta, r, \theta', r')$ to be evenly spread in the delay and angular dimensions. Here, we minimize the p-norm of the ambiguity function $\chi(\tau, \upsilon, \theta, r, \theta', r')$. Mathematically, the corresponding optimization problem can be expressed as

$$\begin{aligned} &\min_C F_p(C) \\ &\text{s.t.} \pm C \in \{1, \cdots, K\}^{M \times 1} \\ &\qquad \Delta f_m \neq \Delta f'_m, \text{for } m \neq m' \end{aligned} \quad (10)$$

where $F_p(C)$ is expressed in (11), and Δf_m, $m = 1, 2, \cdots, M$ denotes the entry of the vector C. Note that a greater p imposes more penalty on the higher peaks. The feasible set of this problem is a discrete set. It is well known that the simulated annealing algorithm is suitable for solving this kind of problems [24].

Simulated annealing algorithm will have a high probability to require a small cost function output by running the MCMC and gradually decreasing the temperature T. In this paper, from state C to C', the transition probability can be expressed as Notes that $C' \leftarrow C$ represents that C' consists transformed from C which only change one element, D denotes $|\{C' | C' \leftarrow C\}|$.

IV. Simulation Results

We consider a uniform linear FDA with 16 elements spaced by d for the simulations. The FDA system operating frequency is 10 GHz and the frequency increment vector C is randomly generated by the use of K = 100 KHz $\cdots$ 10000 KHz. $r_{\min}$ = 10 km, $r_{\max}$ = 15 km, $R_{\min}$ = 10 km , $R_{\max}$ = 20 km, $\theta_{\min} = -\pi/3$ and $\theta_{\max} = \pi/3$ are chosen for the cost function (10). Figure 2 shows a conventional phased-array radar ambiguity function when $\theta = \pi/6$. It is clearly seen that the phased-array radar has range-Doppler coupled and defocused ambiguity function, which implies phased-array

$$\begin{aligned}&\chi\left(\tau,\upsilon,\theta,r,\tau',\upsilon',\theta',r'\right)\\&=\int_{-\infty}^{\infty} y^H\left(t,\tau',\upsilon',\theta',r'\right)y\left(t,\tau,\upsilon,\theta,r\right)dt\\&=M\times\sum_{m=1}^{M}\sum_{m'=1}^{M}\left\{\exp\left\{j2\pi\left(\begin{array}{c}T_0\left(\Delta f_m-\Delta f_{m'}\right)-\frac{\left(\Delta f_m r-\Delta f_{m'}r'\right)}{c_0}\\+\frac{f_0 d\left((m-1)\sin\theta-(m'-1)\sin\theta'\right)}{c_0}\end{array}\right)\right\}\chi_{m,m'}\left(\tau,\upsilon\right)\right\}\end{aligned} \tag{8}$$

$$F_p\left(C\right)=\frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\theta_{\min}}^{\theta_{\max}}\int_{\theta_{\min}}^{\theta_{\max}}\int_{r_{\min}}^{r_{\max}}\int_{r_{\min}}^{r_{\max}}\left|\chi\left(\tau,\upsilon,\theta,r,\theta',r'\right)\right|^p dr dr' d\theta d\theta' d\upsilon d\tau}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{R_{\min}}^{R_{\max}}\int_{R_{\min}}^{R_{\max}}\left|\chi\left(\tau,\upsilon,\theta,r,\theta',r'\right)\right|^p dr dr' d\theta d\theta' d\upsilon d\tau} \tag{11}$$

$$p(C,C')=\begin{cases}\frac{1}{D}min\left(1,\exp\left(\left(F_p\left(C\right)-F_p\left(C'\right)\right)/T\right)\right), & \text{if}\pm C'\leftarrow C\\ 1-\frac{1}{D}\sum_{C'\sim C}min\left(1,\exp\left(\left(F_p\left(C\right)-F_p\left(C'\right)\right)/T\right)\right), & \text{if}\pm C'=C\\ 0, & \text{otherwise}\end{cases} \tag{12}$$

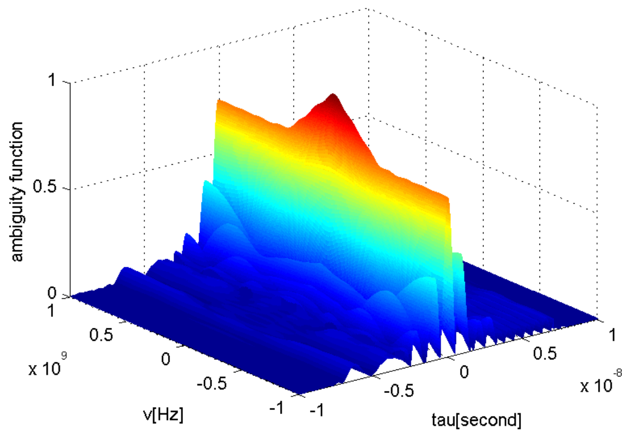

Fig. 2. Three-dimensional ambiguity function illustration of phased-array radar.

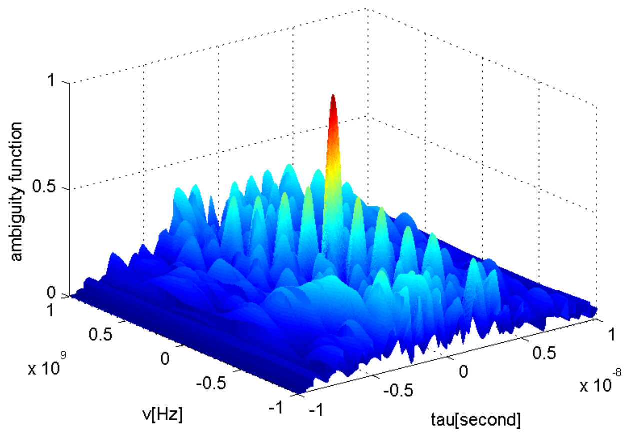

Fig. 3. Three-dimensional ambiguity function illustration of non-optimized FDA radar.

radar cannot effectively distinguish range-angle dependent targets.

Figure 3 shows the radar ambiguity function of a non-optimized FDA radar. Despite the ambiguity function accumulates into a spike, but the sidelobe peak of the ambiguity function is still relatively large which may bring false detection of targets. In contrast, when the proposed optimization algorithm is applied, the ambiguity function performance is improved because the sidelobes have been significantly suppressed, as showed in Figure 4. Table 1 compares the ratio between the maximum sidelobe peak and the mainlobe peak, referred as SPR, which validates also that performance improvements have been obtained by the optimization algorithm. This means that the optimized FDA radar will have more robust capability to identify targets.

V. Conclusions

This paper derives the FDA radar ambiguity function in a general way. Then a simulated annealing algorithm is

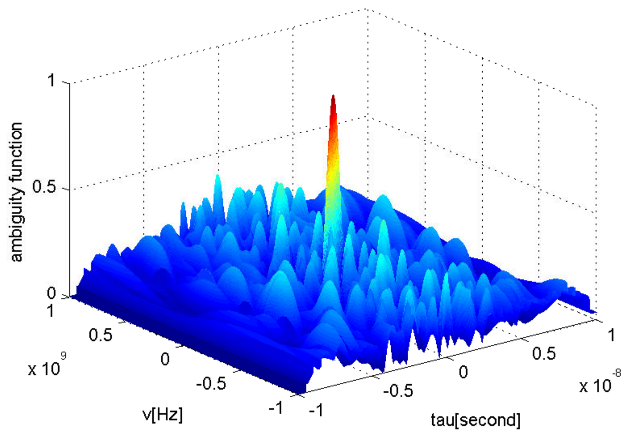

Fig. 4. Three-dimensional ambiguity function illustration of optimized FDA radar.

TABLE I
COMPARISONS OF SPR PERFORMANCE WHEN DIFFERENT ARRAY PARAMETERS ARE EMPLOYED.

parameters		Non-optimized FDA radar	Optimized FDA radar
θ	r/km	SPR	SPR
$\pi/9$	11	0.5532	0.4274
$\pi/3$	11	0.5450	0.4444
$\pi/6$	11	0.5421	0.4832
$\pi/6$	10	0.4911	0.4594

proposed to design the FDA radar through optimizing the ambiguity function which directly relates to the FDA radar system performance. Simulation results show that the proposed method can obtain lower sidelobe level without degrading the mainlobe performance. Since narrow-band transmission is assumed in this paper, we plan to further investigate the wide-band transmission in future work.

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