Compressive Sensing-Based Range and Angle Estimation For Nested FDA Radar

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***Abstract*—This paper proposes a nested frequency diverse array (FDA) design scheme, which jointly utilizes FDA angle-range-dependent beampattern and nested array increased degrees-of-freedom (DOFs). In this case, the commonly used MUSIC algorithm fail to estimate multiple sources because of the source correlation. Furthermore, a compressive sensing-based range and angle estimation algorithm is proposed. While compared with traditional spatial smoothing (SS) based localization method, our proposed method requires less snapshots and better performance.**

I. INTRODUCTION

Different from conventional phased-array antenna [1-3], frequency diverse array (FDA) [4] employs a small amount of frequency increment across the array elements, instead of a linear phase shifting in the phased array antenna. Because of the frequency increment, FDA has range and angle dependent beampattern. This provides a potential applications in joint range and angle estimation [5]. In contrast, phased-array radar can estimate only the angle [6].

Meanwhile, a new array geometry named nested array [7], which is capable of significantly increasing the array degree of freedom (DOF), has been proposed as an attractive technique to construct sparse arrays. Its structure can be obtained by systematically nesting two or more uniform linear arrays and provide $O(N^2)$ DOF when N physical sensors and the second-order statistics of the received data are exploited. However, most of pioneer works focus on receiving nested array, In this paper, we present a nested FDA transmitter, which data are processed by the difference coarray algorithm. In this case, the traditional spatial smoothing (SS) [8] is often employed to decorrelate the incoming coherent signals, but it will significantly reduce the available DOFs of the system. To fully exploit the DOF in the nested FDA [9], the compressive sensing (CS) technique is exploited to both range and angle estimation. Compared with conventional SS based method, the proposed technique not only takes less snapshots but also results in better performance.

The manuscript is organized as follows. The nested FDA is presented in Section II. The CS-based range and angle reconstruct algorithm is presented in Section III. Next, numerical results are provided in Section IV to evaluate the proposed method. Finally, This paper is concluded in Section V.

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II. NESTED FDA SIGNAL MODEL

In a standard uniform linear array (ULA) FDA, the waveforms radiated from all elements are identical except with a frequency increment Δf Hz. That is, the radiated frequency of the mth element is

$$f_m = f_0 + m \cdot \Delta f, \quad m = 0, 1, \ldots, M-1 \tag{1}$$

where f_0 is the radar carrier frequency and M is the number of the array elements.

When the beam is steered to the angle θ and range r, the transmitted signal arriving at the point can be expressed as [10]

$$s(\theta, r, t) = \sum_{m=0}^{M-1} \exp\left\{-j2\pi f_m \left(t - \frac{r}{c_0} + \frac{md\sin\theta}{c_0}\right)\right\} \tag{2}$$

where c_0 is the speed of light and d is element spacing. Note that uniform real weighting (all ones) is assumed.

For a given time reference, e.g., $t = 0$, (2) can be converted to

$$\begin{aligned} p(\theta, r) &= \sum_{m=0}^{M-1} \exp\left\{j2\pi(f_0 + m\Delta f)\left(\frac{r}{c_0} - \frac{md\sin\theta}{c_0}\right)\right\} \\ &= A_0(r) \sum_{m=0}^{M-1} \exp\left\{j\frac{2\pi}{c_0}(m\Delta f r - mf_0 d\sin\theta - m^2\Delta f d\sin\theta\right\} \\ &\approx A_0(r) \sum_{m=0}^{M-1} \exp\left\{j\frac{2\pi}{c_0}(m\Delta f r - mf_0 d\sin\theta)\right\} \end{aligned} \tag{3}$$

where $A_0(r) = \exp\{j2\pi f_0 r/c_0\}$.

According to (3), the beampattern achieves its maxima at

$$m\Delta f r - mf_0 d\sin\theta = l, \quad l = 1, 2, \ldots \tag{4}$$

This implies that the FDA radar can offer a range-angle-dependent beampattern. A nested FDA model is shown in Fig.1, a 2-level ULA is described, where different distances and frequency increments are used for this structure, such as frequency increment Δf with distance d is used in level 1

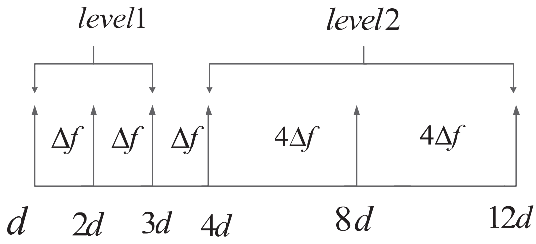

Fig. 1: A 2 level nested array with 3 sensors in each level

while frequency increment $4\Delta f$ with distance $4d$ is used in level 2.

It is proved that, the maximum DOF that can be obtained from an total element array N with any geometry [7] is

$$DOF_{\max} = N(N-1)+1. \tag{5}$$

Consider a multiple output single input (MOSI) FDA system in Fig.2 which has m transmit antennas and one receive antenna. The received signal consists of all the echoes transmitted

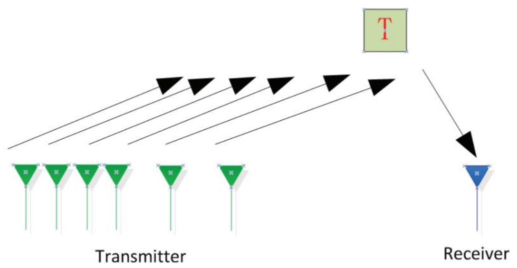

Fig. 2: A MOSI FDA system

from the array elements, and can be expressed as

$$r(\theta,r,t) = \sum_{m=1}^{M} \exp\{j2\pi f_m(t-\tau'_m)\} = \sum_{m=1}^{M} \exp\{j2\pi f_m(t-(\tau_m+\tau))\} \tag{6}$$

where τ'_m donate the whole delay from the transmit to receiver, and τ_m is the delay from the transmit to the target, and τ is the delay from the target to the receiver, because only one received is used, so τ is a scalar while τ_m is a vector. (6) can be further simplified as

$$\begin{aligned} r(\theta,r,t) &= \sum_{m=1}^{M} \psi_m(t)\exp\{j2\pi f_m(t-\tau_m)\}\exp\{j2\pi f_m(-\tau)\} \\ &= \exp(j\varphi_0)\psi_m(t)\mathbf{a}(\theta,r,t)\mathbf{b}(\theta,r,t) \\ &= \exp(j\varphi_0)\psi_m(t)\mathbf{a}'(\theta,r,t)\mathbf{w}(\theta,r,t) \end{aligned} \tag{7}$$

where $\psi_m(t)$ is the baseband waveform for the m-th subarray. $\mathbf{a}(\theta,r,t)$ and $\mathbf{b}(\theta,r,t)$ denotes the transmiting and receiving steering vector respectively. The synthesis steering vector $\mathbf{a}'(\theta,r,t) = [1 \quad \exp\{-j\varphi_{r1}\} \quad \cdots \quad \exp\{-j\varphi_{rM-1}\}]$ is with $\varphi_{rm} = 2\pi(f_0 md\sin\theta/c + 2m\Delta f r/c)$, and $\mathbf{w} = [1 \ \ 1 \ \ \ldots \ \ 1]$.

Because the waveform radiated from each antenna has different frequency, according to [11], after matching filter banks, the received data can be expressed as

$$\mathbf{x} = \sqrt{\frac{E}{N}}\xi\mathbf{a}'(\theta,r,t) \tag{8}$$

where $\sqrt{\frac{E}{N}}$ denotes signal-to-noise ratio, and ξ is the complex valued reflection coefficient of the target.

$$\begin{aligned} R_{xx} &= E[xx^H] = \mathbf{a}'(\theta,r,t)R_{\xi\xi}\mathbf{a}'(\theta,r,t)^H + \sigma_n^2 I \\ &= \mathbf{a}'(\theta,r,t)\begin{pmatrix} \sigma_1^2 & & & \\ & \sigma_2^2 & & \\ & & \ddots & \\ & & & \sigma_D^2 \end{pmatrix}\mathbf{a}'(\theta,r,t)^H + \sigma_n^2 I \end{aligned} \tag{9}$$

When D narrowband sources are impinged on this array from different locations $\{(\theta_i\ ,R_i)i = 1,2,...,D\}$ with powers $\{\sigma_i^2, i = 1,2,...,D\}$. In the following, time t is fix to a constant variable.

III. RANGE AND ANGLE ESTIMATION

A. Traditional Spatial Smoothing Based Algorithm

The data model is the similar as [12]. The difference is the steering vector $\mathbf{a}'(\theta,r)$ is not only the function of θ , but also the function of r. Now, R_{xx} in (9) is vectorized to the following vector:

$$z = vec(R_{xx}) = (\mathbf{a}'^H \odot \mathbf{a}')p + \sigma_n^2\tilde{i} = \tilde{A}p + \sigma_n^2\tilde{i} \tag{10}$$

where $p = [\sigma_1^2 \ \ \sigma_2^2 \ \ \ldots \ \ \sigma_n^2]^T$ and $\tilde{i} = [e_1^T \ \ e_2^T \ \ \ldots \ \ e_N^T]^T$ with e_i^T being a column vector of all zeros except a 1 term at the ith position. The function of z in (10) behaves like the received signal at an array whose manifold is given by $\mathbf{a}'^H \odot \mathbf{a}'$, where $\odot$ denotes the KR product. The equivalent source signal vector is represented by p and the noise becomes a deterministic vector given by $\sigma_n^2 e_n$.

Define

$$R_i = E[zz^H] \tag{11}$$

To apply spatial smoothing [10], a new autocorrelation matrix R_{ss} can be used as

$$R_{ss} \triangleq \frac{1}{(\frac{N^2}{4}+\frac{N}{2})}\sum_{i=1}^{N^2/4+N/2} R_i \tag{12}$$

We call the matrix R_{ss} as the spatially smoothed matrix.

$$\begin{aligned} R_{ss} &\triangleq \mathbf{a}'(\theta,r)\begin{pmatrix} \Lambda & 0 \\ 0 & 0 \end{pmatrix}\mathbf{a}'(\theta,r)^H + \sigma_n^2 I \\ &= [U_A \ U_n]\begin{pmatrix} \Lambda + \sigma_S^2 I & 0 \\ 0 & \sigma_{M-s}^2 I \end{pmatrix}[U_A \ U_n]^H \end{aligned} \tag{13}$$

where U_A and U_n are the unitary matrixes of signal and noise subspaces, respectively, Λ denotes the target signal power, $\Lambda + \sigma_S^2 I$ is the signal covariance and $\sigma_{M-s}^2 I$ is regarded as the noise covariance. Hence by applying MUSIC on R_{ss}, the targets can be localized by searching the peaks:

$$\{\theta, R\} = \max\{\frac{1}{\mathbf{a}'(\theta,r)U_n U_n^H \mathbf{a}'(\theta,r)^H}\}. \tag{14}$$

B. Proposed Compressive Sensing Based Algorithm

From (10), we find out that, p is a vector which is constructed by targets energy. When the targets are detected, the terms in p have minimum zero value. Then we consider solving (10) in terms of the sparse signal recovery in compressive sensing theory. The desired result of p is represented as the solution to the following constrained minimization problem:

$$\hat{p} = \arg\min_{p} \|p\|_0 \quad s.t.\|z - \tilde{A}p - \sigma_n^2 \tilde{i}\|_2 < \xi \tag{15}$$

where ξ is a user-specific bound.

For convenience, we rewrite the constraint function as

$$\left\|z - \tilde{A}p - \sigma_n^2 \tilde{i}\right\|_2 = \left\|z - (\tilde{A}p + \sigma_n^2 \tilde{i})\right\|_2 = \|z - Br\|_2 \tag{16}$$

where $B = [\tilde{A}, \tilde{i}]$ and $r = [p^T, \sigma_n^2]^T = [\sigma_1^2, ..., \sigma_D^2, \sigma_n^2]$. Then, the above expression can be redefined as

$$\hat{r} = \arg\min_{r} \|r\|_0 \quad s.t.\|z - Br\|_2 < \xi \tag{17}$$

Because l_0 norm is a NP hard problem, so we use the batch Lasso method [13], the l_0 norm in the above expression is replaced by the l_1 norm.

$$\hat{r} = \arg\min_{r} \|r\|_1 \quad s.t.\|z - Br\|_2 < \xi \tag{18}$$

Now, this problem is converted to convex optimization. Herein, we can use the effective tool CVX to solve this problem.

IV. SIMULATION RESULTS

In this section, several numerical examples are provided to illustrate the superior performance of the CS based range and angle estimation for the nested FDA. In the examples, A 2-level nested array with $N = 3$ sensors in each level is used. The array has maximum of 12DOFs. Assuming $D = 10$ sources which angles are distributed from -50^0 to 50^0 with an interval of 11^0 and the ranges are from $7.5km$ to $12.5km$ with an interval of $520m$. The other parameter values are: $SNR = 0dB$, $\Delta f = 55kHz$ and $f_0 = 10GHz$.

A. Angle and Range estimation

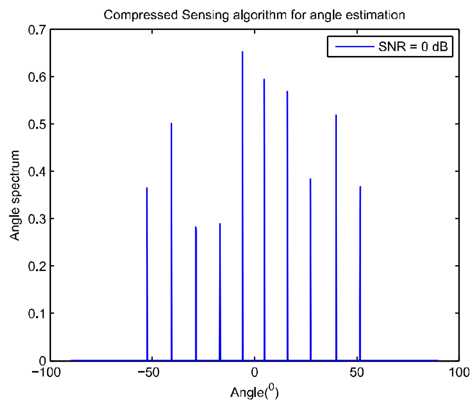

Fig. 3: Compressed sensing for angle estimation of targets

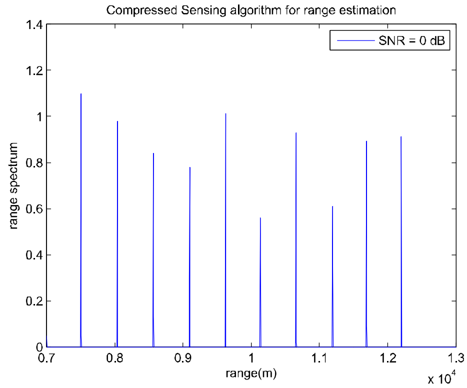

Fig. 4: Compressed sensing for range estimation of targets

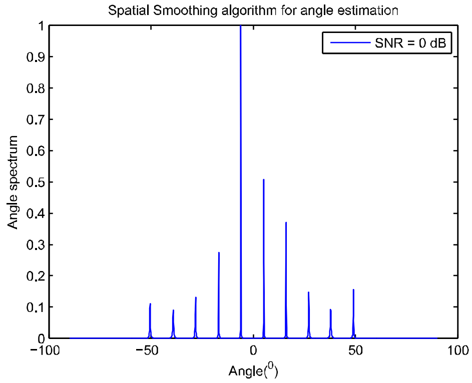

Fig. 5: Spatial smoothing for angle estimation of targets

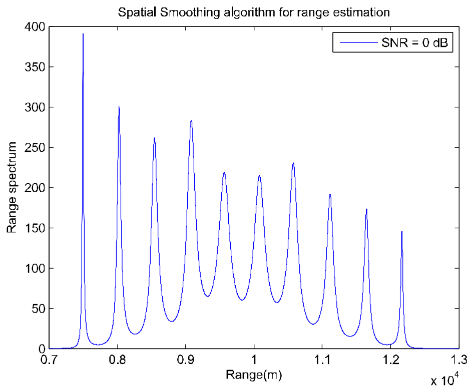

Fig. 6: Spatial smoothing for range estimation of targets

Fig.3-4 indicate the angle and range estimation of targets by CS based method, all the 10 targets can be found. For comparison, angle and range estimation of targets by SS based method are shown in Fig.5-6. Resolution of SS based method is obviously lower than that of proposed CS method in both angle and range estimation.

B. RMSE versus SNR

We compare the SS and CS based method by studying RMSE vs SNR performance. Both RMSE of range and angle estimation are shown below.

Fig.7 shows that the SS based angle estimated performance is worse than CS based one in low SNR with snapshot =1000.

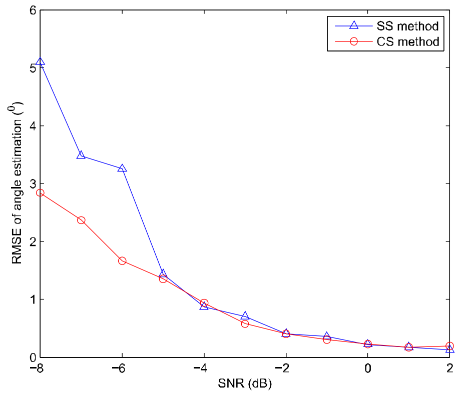

Fig. 7: Angle RMSE vs SNR with snapshot = 1000

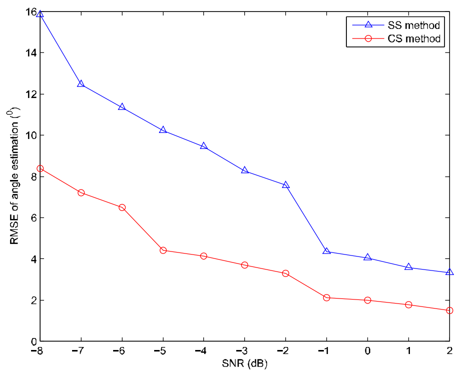

Fig. 8: Angle RMSE vs SNR when snapshot = 100

But when snapshot is decreased to 100, the CS based method is obviously better than the SS based one in Fig.8, that indicates the CS based method can work well under the less snapshot.

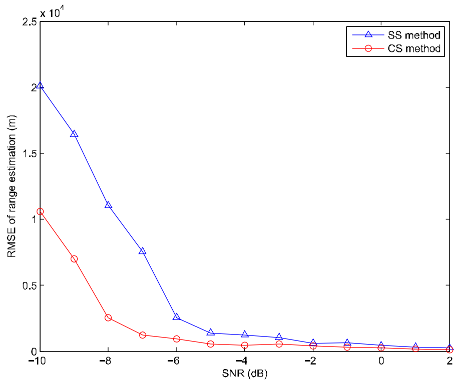

Fig. 9: Range RMSE vs SNR with snapshot = 1000

Similarly, Fig.9 shows the performance of the range estimation. CS based method is just better than SS based method in low SNR with snapshot =1000. However, when snapshot is set to 100 in Fig.10, CS based method outperforms the SS based method in both low and high SNR.

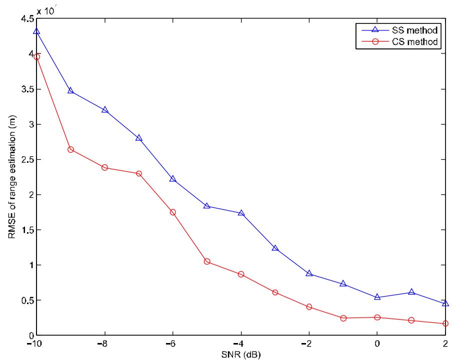

Fig. 10: Range RMSE vs SNR when snapshot = 100

V. Conclusion

In this paper, we jointly exploited FDA and nested array structure in transmitter for exploiting the multiple DOF. Moreover, the CS based method is employed for target range and angle estimation. Numerical simulation results demonstrate that, compared with traditional SS based localization method, our proposed method requires less snapshots and gets better performance.

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